

Actuarial Society of South Africa

EXAMINERS REPORTS

MAY 2021

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

Question 1

- **The objectives of the modelling exercise.** Clear objective, predicting number of covid-19 cases.
- **The validity of the model for the purpose to which it is to be put.** While the S-I-R model may have been used for other epidemics in the past, it may not be very useful in the case of COVID-19 where factors like migration seem to have a significant impact.
- **The validity of the data to be used.** There is a delay between when data are recorded and when data are available for use. The model is being developed at the beginning of an epidemic and so the availability and validity of data may be problematic.
- **The validity of the assumptions.** Constant population assumption unrealistic.
- The **possible errors** associated with the model or parameters used not being a perfect representation of the real-world situation being modelled. Interaction between the categories is stipulated as α_1, α_2 and α_3 . This may be incorrect type of parameter or value of parameter incorrect.
- The **impact of correlations** between the random variables that drive the model. Interaction between the categories is stipulated as α_1, α_2 and α_3 and this may be incorrect.
- The **extent of correlations** between the various results produced from the model. An increase in the number of people infected at any point in time, should lead to a subsequent increase in the number of people in R.
- **The current relevance of models written and used in the past.** There are no past COVID-19 models to compare with, so how relevant is suggested model?
- **The credibility of the data input.** The causes of deaths are often not accurately recorded. And some health facilities may not have necessary equipment to test sick individuals for the virus. This is most common in developing countries like South Africa.
- **The credibility of the results output.** The output of the model should be checked against actual cases over time (as the pandemic develops)
- **The dangers of spurious accuracy.** There are high chances of spurious accuracy derived from prescribing a model like the S-I-R with limited or little data.
- **The ease with which the model and its results can be communicated.** Simple model structure, so easier to communicate to non-experts.
- **Regulatory requirements.** Human ethics clearance if conducted through universities.

[Total 7]

Candidates are still missing out on easy theory marks which makes a difference to the pass/fail grade of marginal candidates.

This question required candidates to think critically about the model and any sensible comments were given credit. Candidates must be careful of only giving positive feedback on the model, as the examiners wish to see candidates evaluate the model critically.

Question 2

i.

$$\begin{aligned} \left(1 + \frac{i^{(p)}}{p}\right)^p &= \left(1 - \frac{d^{(p)}}{p}\right)^{-p} \\ \Rightarrow \frac{p}{p + i^{(p)}} &= 1 - \frac{d^{(p)}}{p} \\ \Rightarrow \frac{d^{(p)}}{p} &= \frac{i^{(p)}}{p + i^{(p)}} \\ \Rightarrow \frac{p}{d^{(p)}} &= \frac{p + i^{(p)}}{i^{(p)}} \\ \Rightarrow \frac{1}{d^{(p)}} &= \frac{1}{i^{(p)}} + \frac{1}{p} \end{aligned}$$

[Total 4]

ii.

$\frac{1}{d^{(p)}} = \ddot{a}_{\infty|}^{(p)}$ is the present value of a perpetuity, payable p times per annum, in advance with a payment of $\frac{1}{p}$ payable every period of length $\frac{1}{p}$

$\frac{1}{i^{(p)}} = a_{\infty|}^{(p)}$ is the present value of a perpetuity, payable p times per annum, in arrears with a payment of $\frac{1}{p}$ payable every period of length $\frac{1}{p}$

The difference between $\ddot{a}_{\infty|}^{(p)}$ and $a_{\infty|}^{(p)}$ is the payment at $t=0$ thus $\frac{1}{p}$

$$\begin{aligned} \ddot{a}_{\infty|}^{(p)} - a_{\infty|}^{(p)} &= \frac{1}{p} \\ \Rightarrow \frac{1}{d^{(p)}} &= \frac{1}{i^{(p)}} + \frac{1}{p} \end{aligned}$$

[Total 3]

There are alternative ways to prove part (i).

Part (ii) was the worst answered question in the paper. Candidates must always follow the instructions in the questions. In this question the command verb was “explain” and the examiners expected an explanation in words.

Despite the fact that the information was provided in the question, candidates still failed to recognize that $\frac{1}{d^{(p)}}$ and $\frac{1}{i^{(p)}}$ represented the present value of a perpetuity, payable p times per annum in advance and in arrears, respectively.

Question 3

- Bond yields are determined by investor's expectations of future movements in interest rates so that returns from bonds of different terms to maturity are equal to expected returns from a series of short-term investments.
- The liquidity preference theory reflects a preference for shorter-term investments as they are less volatile thus long-term investments need to provide a yield to maturity greater than that implied by expectations theory.
- Market segmentation theory reflects supply and demand conditions in particular segments of the market (short, medium and long term).

[Total 4]

Question 4

i.

$$PV = 5 \int_0^8 e^{0.05t} \times \exp\left(-\int_0^t 0.03ds\right) dt + 5 \int_8^{15} e^{0.05t} \times \exp\left(-\int_8^t 0.05ds\right) dt \times \exp\left(-\int_0^8 0.03ds\right)$$

Part 1: $5 \int_0^8 e^{0.05t} \times \exp\left(-\int_0^t 0.03ds\right) dt = 5 \int_0^8 e^{0.05t} \times \exp\left(-0.03s\Big|_0^t\right) dt$

$$= 5 \int_0^8 e^{0.05t} \times \exp(-0.03t) dt = 5 \int_0^8 e^{0.02t} dt = \frac{5}{0.02} e^{0.02t} \Big|_0^8$$

$$= 250(e^{0.16} - 1) = 43.37771775$$

Part 2: $5 \times \int_8^{15} \left(e^{0.05t} \times \exp\left(-\int_8^t 0.05ds\right) \right) dt \times \exp\left(-\int_0^8 0.03ds\right)$

$$\begin{aligned}
&= 5 \times \int_8^{15} \left(e^{0.05t} \times \exp(-0.05s|_8^t) \right) dt \times \exp(-0.03s|_0^8) \\
&= 5e^{-0.24} \times \int_8^{15} e^{0.05t} \times e^{-0.05(t-8)} dt \\
&= 5e^{-0.24} \times e^{0.4} \times \int_8^{15} 1 dt \\
&= 5 \times e^{0.16} \times t|_8^{15} \\
&= 5 \times e^{0.16} \times (15-8) = 41.07288048
\end{aligned}$$

Total PV:

$$= 43.37771775 + 41.07288048 = 84.45059823$$

[Total 10]

ii.

$$\text{a. } (1.025)^2 = \left(1 - \frac{d^{(3)}}{3}\right)^{-3} \Rightarrow d^{(3)} = 4.8980963\%$$

$$\text{b. } (1.025)^{2 \times 4} = \left(1 + \frac{i^{(\frac{1}{4})}}{\frac{1}{4}}\right)^1$$

$$1.218402898 = \left(1 + i^{(\frac{1}{4})} \times 4\right)^1 \Rightarrow i^{(\frac{1}{4})} = 5.4600724\%$$

[Total 4]

In part (i) the common mistake was not splitting the integral into two parts. This was necessitated by the change in the $\delta(t)$ at time 8. Despite the question not being too technically complex, these candidates missed out on much of the credit due to this fundamental error.

Examiners expect candidates to show their working when doing the integration. The model solution indicates the steps that are needed for full credit.

Part (ii) was answered well.

Question 5

i.

(Work in millions)

We have $PV_L(i) = 100v^{10} + 200v^{15}$ @ $i = 0.07$ and $v = 0.934579$

Therefore $PV_L(7\%) = 123.324 = 123.32$

Also $PV_A(i) = 74.04v^2 + 318.34v^{25} \Rightarrow PV_A(7\%) = 123.323 = 123.32$

Then $PV_L(7\%) = PV_A(7\%)$.

The positive derivatives of assets and liabilities give:

$$PV'_L(i) = 1000v^{11} + 3000v^{16} \Rightarrow PV'_L(7\%) = 1491.3$$

$$PV'_A(i) = 148.08v^3 + 7958.5v^{26} \Rightarrow PV'_A(7\%) = 1491.3$$

$$\Rightarrow Vol_L(7\%) = \frac{1491.3}{123.324} = 12.0926 \quad \text{and}$$

$$\Rightarrow Vol_A(7\%) = \frac{1491.3}{123.323} = 12.0925 = Vol_L(7\%)$$

Therefore, the first 2 conditions are satisfied correct to 2 decimal places.

[Total 7]

ii.

If the interest rate increases to 8% then $PV_L(8\%) = 109.3676898$ and

$$PV_A(8\%) = 109.9607061$$

$$\text{Therefore Profit} = 109.9607061 - 109.3676898 = R593,016.3035$$

[Total 2]

iii.

- The DMT = 12.939 years for both assets and liabilities
- the spread of the assets is greater than the spread of the liabilities around the DMT
- $\Rightarrow$ Redington's third condition for immunization is also satisfied
- $\Rightarrow$ profit will occur if there is a small change in the rate of interest

Part (i) and (ii) was answered well. Candidates should note that when a question asks specifically for Redington's Theory the second condition must always be stated in terms of the volatility.

Part (iii) was poorly answered as many candidates

- either assumed the fund was immunized or
- explained the profit/loss in terms of the increase in the interest rates.

Either of these errors could have been avoided by reading the question carefully and following the instructions. The command verb was "explain" - thus candidates should not make an assumption – and the question clearly stated "with reference to Redington's Theory of immunisation" – thus candidates should set out their answer in terms of immunisation.

Stronger candidates were able to methodically separate out the distinct points and present the argument in a logical manner.

Question 6

i.

We have $i^{(2)} = 0.08$. Thus $i^{(12)} = 0.078698363$

Therefore $PV = 3,600a_{\overline{25}|i^{(12)}}^{(12)} = 300a_{\overline{25 \times 12}|i^{(12)}}^{(12)} = 3,600 \times 10.91874531 = R39,307.48312283$

[Total 3]

ii.

The outstanding balance after the 120th payment is

$$L_{120} = 39,307.48312283 \times \left(1 + \frac{i^{(12)}}{12}\right)^{120} - \left(3,600s_{\overline{10}|i^{(12)}}^{(12)} + 1,000\left(1 + \frac{i^{(12)}}{12}\right)^{36} + 1,000\right)$$

$$= R29,375.145825007$$

[Total 4]

iii.

Let X be the new annual repayment.

Therefore, the equation of value is $29,400 = Xa_{\overline{8}|i^{(12)}}^{(12)} @$

$$i = 7\% \Rightarrow i^{(12)} = 0.067849745 \quad \frac{i^{(12)}}{12} = 0.005654145$$

$$\Rightarrow 6.16054X = 29,400 \quad \Rightarrow X = 4,772.310867.$$

The monthly repayment is thus $\frac{X}{12} = R397.69257227$.

The 180th payment from the beginning is equal to the 60th payment from the 10th year. We therefore want the interest portion of the 60th payment in the second annuity.

$$\text{We have } L_{59} = 29,400 \left(1 + \frac{0.0678497}{12} \right)^{59} - 397.69257227 s_{\overline{59}|i^{(12)}/12} = 13,243.7674647382$$

$$\text{with } \frac{i^{(12)}}{12} = 0.00565415.$$

Therefore, the interest portion of the 180th payment is

$$\frac{i^{(12)}}{12} \times L_{59} = 0.00565415 \times 13,243.7674647382 = R74.88218672$$

[Total 6]

Part (i) was the best answered question of the paper.

Although the loan question often offers easy marks a large proportion of candidates struggled with part (ii) and (iii).

In part (ii) the retrospective method for calculation the loan outstanding was the most sensible method although the prospective method also calculated the correct answer if the additional payments were correctly allowed for.

In part (iii) the most common errors were

- *calculating L_{60} instead of L_{59} and*
- *not reducing the outstanding term to eight years when the new payment was calculated and/or failing to account for the reduced interest rate applicable over the remaining term.*

Question 7

i.

Let X be the annual income provided by the annuity. Then $Xa_{\overline{5}|i=6.25\%} = 10,000$
 $X = R2,390.132069780$

[Total 2]

ii.

For **Investment A**, the five-year effective interest rate is

$$1.035 \times 1.045 \times 1.055 \times 1.065 \times 1.075 = 1.306372928$$

The five-year growth in RPI is $\frac{275.6}{240} = 1.148333333$

Therefore, the annual real rate of interest is the value of i from

$$1.306372928 \times \frac{240}{275.6} = 1.137625191$$

$$(1+i)^5 = 1.137625191 \Rightarrow i = 2.6123987\%$$

For **Investment B**, the monetary redemption payout is

$$10,000 \times \frac{274}{237.6} = 10,000 \times 1.153199 = 11,531.98653$$

Simple interest: $10,000 \times 5 \times 0.0275 = 1,375$

Total investment @ $t=5$: $11,531.98653 + 1,375 = 12,906.98653$

Thus, the real redemption payment is $12,906.98653 \times \frac{240}{275.6} = 11,239.75605$

Thus, the real rate for B is i such that

$$(1+i)^5 = \frac{11,239.75605}{10,000} = 1.123975605 \Rightarrow i = 2.3649732\%$$

[Total 9]

iii.

The real equation of value is

$$10,000 = 2,390.13 \left(\frac{240}{250} v + \frac{240}{264.4} v^2 + \frac{240}{266.6} v^3 + \frac{240}{270.4} v^4 + \frac{240}{275.6} v^5 \right) \quad @i = \text{real yield}$$

Sub $i = 2.722\%$ into RHS = 10,000.12887

[Total 6]

Part (i) was well answered by the majority of candidates.

Part (ii) and (iii) were some of the worst answered questions in the paper. The topics of

- *real interest rates and*
- *how to use the Retail Price Index (RPI)*

continues to be problematic for many candidates.

In part (ii) and (iii) the common errors were

- *ignoring inflation altogether*
- *using RPI of the incorrect dates*
- *inverting the RPI*
- *Investment B: adding inflation to the simple interest.*

When candidates are asked to “Show that an annual rate of return is approximately equal to”, the most efficient way is to substitute the given rate of return into the right hand side of the equation and show that it is equal to the left hand side of the equation.

A valid alternative (but much more time consuming) approach would be to do linear interpolation but candidates must show all the workings for the linear interpolation to obtain full credit.

Question 8

i.

- The investor has an initial known negative cashflow.
- The size of the positive payments is unknown because the company that issued the share may not declare dividends and thus no payment may be made.
- The timing of the dividend payments is also unknown as dividends may be held back or not paid subject to the profitability of the company owning the share.
- There is usually unknown redemption date for the share as the date of sale is unknown thus a final positive cashflow is uncertain in terms of magnitude and timing.

[Total 4]

ii.

$$P = D(1+g)v_i + D(1+g)^2 v_i^2 + D(1+g)^3 v_i^3 + \dots$$

$$\frac{1}{(1+j)} = \frac{(1+g)}{(1+i)} \Rightarrow j = \frac{(1+i)}{(1+g)} - 1 = \frac{i-g}{1+g}$$

$$P = Dv_j + Dv_j^2 + Dv_j^3 + \dots$$

$$P = \frac{D}{j} = \frac{D}{\left[\frac{i-g}{1+g} \right]} = \frac{D \times (1+g)}{i-g}$$

[Total 4]

iii.

a. Let $v = \frac{1}{1.06} = 0.943396$. The value of Company A share is

$$PV_{CompA} = v^6 \times 1.08 + 1.08 \times 1.1 \times v^7 + 1.08 \times 1.1 \times v^8 + \dots + 1.08 \times 1.1^6 \times v^{12}$$

$$+ 1.08 \times 1.1^6 \times (1.03v^{13} + 1.03^2 v^{14} + \dots) \quad @ i = 6\%$$

$$= v^6 \times \left[(1.08 + 1.08 \times 1.1 \times v + 1.08 \times 1.1^2 \times v^2 + \dots + 1.08 \times 1.1^6 \times v^6) \right.$$

$$\left. + 1.08 \times 1.1^6 (1.03 \times v^7 + 1.03^2 \times v^8 + \dots) \right]$$

$$\text{Let } w_1 = \frac{1.1}{1.06} = 1.037735849 \quad ; \quad w_2 = \frac{1.03}{1.06} = 0.971698113 = \frac{1}{1+j} \quad \text{and}$$

$$j = \frac{1}{w_2} - 1 = 0.0291262. \text{ Then}$$

$$PV_{CompA} = 1.08 \times v^6 \times \left(\frac{1-w_1^7}{1-w_1} + 1.1^6 \times v^6 \times a_{\infty}^- \right)$$

$$= 1.08 \times v^6 \times \left(\frac{1-w_1^7}{1-w_1} + 1.1^6 \times v^6 \times \frac{1}{j} \right)$$

$$= 0.761357384 \times (7.84422 + 42.8782) = 38.61791215$$

[Total 9]

$$\begin{aligned}
b. \quad PV_{CompB} &= 0.75v + (0.75 + 0.5) \times v^2 + (0.75 + 2 \times 0.5) \times v^3 + \dots + 29 \times 0.5 \times v^{30} \\
&= (0.25v + 0.25v^2 \dots^{30}) + (0.5 \times v^1 + (2 \times 0.5) \times v^2 + \dots 0.5) \times v^{30} \\
&= 0.25 \times a_{\overline{30}|=6\%} + 0.5 \times (Ia)_{\overline{30}|=6\%} \\
&= 3.441207788 + 78.06180911 = 81.5030169
\end{aligned}$$

[Total 4]

c.

- With an increase in the valuation interest rate, the value of the share (both for company A and B) will decrease.
- The reason is that there is an inverse relationship between the PV and the interest rate.
- The proportionate fall in the value of company A's share will be bigger than the proportionate fall in the value of company B's share
- The reason is that company A's cashflow starts later than company B's
- And continues later than company A
- Thus the change in interest rate has a bigger influence on the value of company A's share

[Total 4]

In part (i) candidates lost mark by answering the question only partially. The question specifically asked to make reference to certainty/uncertainty of cashflow and some candidates ignored this instruction. Stronger candidates were able to distinguish between the certainty/uncertainty of both the amount and timing of the cashflows.

In part (ii) using the geometric series was a valid alternative approach. The command verb in the question is "derive". This implies that each line has to follow logically from the previous line, to obtain full credit.

In part (iii) (a) the most common error was errors in the growth factors for the first 12 years.

Part (iii) (c) required candidates to investigate and contrast the cashflows of the two shares logically and then explain their conclusions. This was poorly done by the majority of candidates.