



ACTUARIAL SOCIETY
OF SOUTH AFRICA

EXAMINATION

2 May 2024 (am)

Subject A211 — Financial Mathematics Intermediate Technical

Time allowed: Two hours and fifteen minutes (which includes 15 minutes of reading time)

Scan and upload time: Twenty minutes (at the end of the examination)

Total marks: 100

INSTRUCTIONS TO THE CANDIDATE

1. Ensure that you are logged in and authenticated through Examity before you attempt the examination.
2. Your computer must be placed, and camera angled, so that your head and shoulders as well as your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your computer by accident.
3. Ensure that you have your exam permit handy. It reflects your candidate number to input **as part of the two hours 15 minutes examination and not before the start of the examination**. Write your candidate number at the top of each page during the examination time only. Do not use your name or member number anywhere on your answer script.
4. The cell phone to be used to scan your final answer script must be switched **OFF** during the two hours and 15 minutes examination time. Place the cell phone at the top of your examination pad / writing pages in view of the invigilator.
5. You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.
6. The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.
7. You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.
8. Attempt all questions, beginning your answer to each question on a new page.
9. Write in black or dark blue pen.

10. *Show calculations where appropriate. You may use blank paper to carry out rough work calculations. This rough work paper must not be scanned as part of your answer script. You may use a calculator from the ASSA-approved list only.*
11. *You may not access any file from your computer, use any other computer program (e.g. Email, MS Word or Excel), or open any other browser during the examination.*
12. *You may not use any other material (e.g. a Formulae and Tables book) during the examination. Any such information that may be required will be provided to you within the examination. **Please refer to the 6%, 8% and 12% tables which have been provided at the end of your question paper.***
13. *Mark allocations are shown in brackets.*
14. *Assume that months are all equal length, unless otherwise stated.*
15. *At the end of the two hours and 15 minutes examination time, you must stop writing and start scanning and uploading your script. **You may NOT continue to write or review your script during this time.***
16. *Scan ALL your answer pages to PDF so that your candidate number is clear at the top of each page.*
17. *Save your PDF scanned file using your candidate number as file name. Do not use your name or member number anywhere in your answer script nor as file name.*
18. *Transfer your scanned script file to your computer and upload into the ASSA Exam Platform.*
19. *Click on the **Upload Answers** link below the examination paper link. Ensure you click on **Finish** below the upload box and again on **Finish All and Submit**, before the 20-minute upload time is up. (After submission the number of files successfully submitted will be reflected.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above.

END OF INSTRUCTIONS

QUESTION 1

Cashflow models are often used to project the sums of money that emerge at different times during a financial transaction.

The following table contains a list of financial products (Column A) with possible cashflow descriptions (Column B).

Column A	Column B
a. Annuity-certain	1. A fixed lump sum at a specified future date and no other positive cashflows.
b. Endowment assurance	2. A contract providing a lump sum benefit payable on death during a specified term.
c. Credit card	3. A contract providing a lump sum benefit at a specified future date and a series of regular level payments in the interim.
d. Zero-coupon bond	4. A contract providing a lump sum benefit payable on survival to the end of a specified term, or on death during the term.
e. Pure endowment	5. A series of regular payments that are dependent upon certain events.
f. Contingent annuity	6. A contract providing a lump sum benefit payable on survival to the end of a specified term.
	7. A flexible borrowing arrangement, with full or partial monthly repayments.
	8. A series of regular payments for a fixed term.
i. Match the financial product in Column A with the most appropriate cashflow description given in Column B by writing down the letter from Column A and the number from Column B (e.g. a – 1).	

[4]

Despite the frequent use of cashflow models in an actuarial context, models are not a simple solution to all actuarial problems.

- ii. Outline eight drawbacks of actuarial models when interpreting model output and communicating the results.

[6]

[Total 10]

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QUESTION 2

You are a junior actuary working for a bank. You have just been approached by an existing client with the following outstanding loans:

Type of loan	Repayment frequency	Annual repayment	Exact remaining term in years	Annual nominal interest rate
Home loan	Monthly in advance	R16,585	12.5	$i^{(2)} = 3.5\%$
Study loan	Quarterly in arrears	R50,000	1.0	$i^{(12)} = 9.5\%$
Personal loan	Monthly in arrears	R60,000	0.5	$i = 16\%$

The client wants to consolidate the outstanding loans into a single repayment schedule at an effective interest rate of 7.5% per annum.

The new consolidated loan is to be repaid continuously at a rate of R25,000 per annum.

Calculate the term of the new repayment schedule.

[Total 14]

QUESTION 3

An actuarial student believes that the n -year spot rate of interest can be modelled with the following equation.

$$\frac{0.05}{(1 - 0.01n)^3}$$

- i. Explain what is meant by the term structure of interest rates.

[1]

Assume that the term structure of interest rates can be modeled by the given formula in all future periods.

- ii. Calculate the price per R100 nominal of an eight-year zero-coupon bond purchased now.

[2]

- iii. Calculate the price per R100 nominal of an eight-year zero-coupon bond purchased in four years' time.

[4]

[Total 7]

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QUESTION 4

A financial institution entered into a contract that requires it to pay R100,000 in exactly ten years' time.

At the outset of the contract, the company decided to immunise this liability against small changes in the interest rate by holding a portfolio of zero-coupon bonds. The company thus purchased an eight-year zero-coupon bond with a nominal value of X and a twelve-year zero-coupon bond with a nominal value of Y .

It has been calculated that $X = 0.64Y$ at the time of immunisation.

- i. Calculate the interest rate applicable to all assets and liabilities at the time of immunisation. [10]

Assume that the institution invested R40,000 in the eight-year bond and R62,500 in the twelve-year bond. The interest rate subsequently changes to 12% per annum effective. The first two conditions of immunisation are met at the new interest rate.

- ii. Determine, by doing the necessary calculations, whether the portfolio is immunised. [5]

- iii. Explain why the above immunisation may not be achievable in practice? [3]

[Total 18]

QUESTION 5

Joyce enrolls for a three-year degree at a university and takes out a loan of R250,000 to fund her studies. The loan is to be repaid in monthly installments over five years. The first installment is due one month after the conclusion of her three-year degree.

The loan is issued at an interest rate of 13% per annum convertible half-yearly.

- i. Calculate the amount of the monthly installment. [4]

Immediately after graduation, Joyce joined a company. At the end of her third year of employment she receives a performance bonus and decides to renegotiate her study loan immediately after paying the 36th installment of the loan. She considers the following two options:

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Option A

Repay the remainder of the study loan with an immediate cash payment of R80,000 and nine monthly payments (in arrears) at an effective annual interest rate of 7%.

Option B

Repay the remainder of the study loan with 18 monthly payments, in arrears, of R10,000 each.

- ii. Calculate the new monthly repayment under Option A. [6]
- iii. Demonstrate whether the APR (annual percentage rate of return), for Option B, is more or less than 5%. [4]

[Total 14]

QUESTION 6

Linda purchased a R100,000 nominal fixed-interest bond at issue. The fixed-interest bond pays quarterly coupons of 10% per annum in arrears and is redeemed at par. The borrower can choose a redemption date exactly eight, nine, or ten years after issue.

Linda purchased the bond at a price which guaranteed her a net redemption yield of at least 8% per annum effective. She is subject to 25% tax on income and 30% tax on capital gains.

- i. Calculate the price at which Linda bought the bond. [8]

After five years, immediately after payment of the coupon then due, Linda sold the bond to Jack at a price that would guarantee Jack a net yield of at least 6% per annum effective. Jack is subject to the same rates of tax as Linda.

- ii. Calculate the price at which Jack bought the bond from Linda. [4]
- iii. Explain whether Linda had to pay capital gains tax when she sold the bond to Jack. [2]

[Total 14]

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QUESTION 7

On 31 December 2023, a company announced that it had earned a profit of R5,000,000 in the 2023 calendar year.

On the same day, the company paid a dividend of R3 per share after which the company's share price closed for the year at R46 per share.

To take advantage of the good publicity, the company offers the following two investment opportunities:

Investment A:

Purchase a 10% share of the company's profits over the next ten years with payments being made annually in arrears. The company's profits are expected to increase by R800,000 per annum over each of the next five years and to increase by a constant amount of RY per annum over each of the subsequent five years.

A constant rate of inflation of 4% per annum effective and an effective real yield of 7.7% per annum is assumed on Investment A.

Investment B:

Purchase a block of 100,000 shares in the company, at R46 per share, the closing share price for 2023.

The price paid for both investment options were equal on 1 January 2024.

- i. Calculate the value of Y for Investment A.

[13]

An investor decided to invest in Investment B. The investor assumed that the annual dividends would increase by 2% in the following year and at 4% per annum thereafter, in perpetuity. The investor further assumed that the constant rate of inflation is 4% per annum.

- ii. Calculate the effective real rate of return per annum on Investment B.

[7]

- iii. Explain, in general, with reasons if it is observed that the internal rate of return of an investment, C, is greater than the internal rate of return of another investment, D, whether that implies that the net present value of C is greater than the net present value of D.

[3]

[Total 23]

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Compound Interest

6%	n	$(1+i)^n$	v^n	s_n	a_n	$(Ia)_n$	$(Da)_n$	n
i	0.060 000	1	1.060 00	0.943 40	1.000 0	0.943 4	0.943 4	1
$i^{(2)}$	0.059 126	2	1.123 60	0.890 00	2.060 0	1.833 4	2.723 4	2
$i^{(4)}$	0.058 695	3	1.191 02	0.839 62	3.183 6	2.673 0	5.242 2	3
$i^{(12)}$	0.058 411	4	1.262 48	0.792 09	4.374 6	3.465 1	8.410 6	4
		5	1.338 23	0.747 26	5.637 1	4.212 4	12.146 9	5
δ	0.058 269	6	1.418 52	0.704 96	6.975 3	4.917 3	16.376 7	6
		7	1.503 63	0.665 06	8.393 8	5.582 4	21.032 1	7
		8	1.593 85	0.627 41	9.897 5	6.209 8	26.051 4	8
$(1+i)^{1/2}$	1.029 563	9	1.689 48	0.591 90	11.491 3	6.801 7	31.378 5	9
$(1+i)^{1/4}$	1.014 674	10	1.790 85	0.558 39	13.180 8	7.360 1	36.962 4	10
$(1+i)^{1/12}$	1.004 868	11	1.898 30	0.526 79	14.971 6	7.886 9	42.757 1	11
		12	2.012 20	0.496 97	16.869 9	8.383 8	48.720 7	12
v	0.943 396	13	2.132 93	0.468 84	18.882 1	8.852 7	54.815 6	13
$v^{1/2}$	0.971 286	14	2.260 90	0.442 30	21.015 1	9.295 0	61.007 8	14
$v^{1/4}$	0.985 538	15	2.396 56	0.417 27	23.276 0	9.712 2	67.266 8	15
$v^{1/12}$	0.995 156	16	2.540 35	0.393 65	25.672 5	10.105 9	73.565 1	16
		17	2.692 77	0.371 36	28.212 9	10.477 3	79.878 3	17
d	0.056 604	18	2.854 34	0.350 34	30.905 7	10.827 6	86.184 5	18
$d^{(2)}$	0.057 428	19	3.025 60	0.330 51	33.760 0	11.158 1	92.464 3	19
$d^{(4)}$	0.057 847	20	3.207 14	0.311 80	36.785 6	11.469 9	98.700 4	20
$d^{(12)}$	0.058 128	21	3.399 56	0.294 16	39.992 7	11.764 1	104.877 6	21
		22	3.603 54	0.277 51	43.392 3	12.041 6	110.982 7	22
$i/i^{(2)}$	1.014 782	23	3.819 75	0.261 80	46.995 8	12.303 4	117.004 1	23
$i/i^{(4)}$	1.022 227	24	4.048 93	0.246 98	50.815 6	12.550 4	122.931 6	24
$i/i^{(12)}$	1.027 211	25	4.291 87	0.233 00	54.864 5	12.783 4	128.756 5	25
		26	4.549 38	0.219 81	59.156 4	13.003 2	134.471 6	26
i/δ	1.029 709	27	4.822 35	0.207 37	63.705 8	13.210 5	140.070 5	27
		28	5.111 69	0.195 63	68.528 1	13.406 2	145.548 2	28
i/d	1.044 782	29	5.418 39	0.184 56	73.639 8	13.590 7	150.900 3	29
$i/d^{(2)}$	1.037 227	30	5.743 49	0.174 11	79.058 2	13.764 8	156.123 6	30
$i/d^{(4)}$	1.032 211	31	6.088 10	0.164 25	84.801 7	13.929 1	161.215 5	31
$i/d^{(12)}$		32	6.453 39	0.154 96	90.889 8	14.084 0	166.174 2	32
		33	6.840 59	0.146 19	97.343 2	14.230 2	170.998 3	33
		34	7.251 03	0.137 91	104.183 8	14.368 1	175.687 3	34
		35	7.686 09	0.130 11	111.434 8	14.498 2	180.241 0	35
		36	8.147 25	0.122 74	119.120 9	14.621 0	184.659 6	36
		37	8.636 09	0.115 79	127.268 1	14.736 8	188.944 0	37
		38	9.154 25	0.109 24	135.904 2	14.846 0	193.095 1	38
		39	9.703 51	0.103 06	145.058 5	14.949 1	197.114 2	39
		40	10.285 72	0.097 22	154.762 0	15.046 3	201.003 1	40
		41	10.902 86	0.091 72	165.047 7	15.138 0	204.763 6	41
		42	11.557 03	0.086 53	175.950 5	15.224 5	208.397 8	42
		43	12.250 45	0.081 63	187.507 6	15.306 2	211.907 8	43
		44	12.985 48	0.077 01	199.758 0	15.383 2	215.296 2	44
		45	13.764 61	0.072 65	212.743 5	15.455 8	218.565 5	45
		46	14.590 49	0.068 54	226.508 1	15.524 4	221.718 2	46
		47	15.465 92	0.064 66	241.098 6	15.589 0	224.757 2	47
		48	16.393 87	0.061 00	256.564 5	15.650 0	227.685 1	48
		49	17.377 50	0.057 55	272.958 4	15.707 6	230.504 8	49
		50	18.420 15	0.054 29	290.335 9	15.761 9	233.219 2	50
		60	32.987 69	0.030 31	533.128 2	16.161 4	255.204 2	60
		70	59.075 93	0.016 93	967.932 2	16.384 5	269.711 7	70
		80	105.795 99	0.009 45	1 746.599 9	16.509 1	279.058 4	80
		90	189.464 51	0.005 28	3 141.075 2	16.578 7	284.973 3	90
		100	339.302 08	0.002 95	5 638.368 1	16.617 5	288.664 6	100

Compound Interest

8%	n	$(1+i)^n$	v^n	s_n	a_n	$(Ia)_n$	$(Da)_n$	n
i	0.080 000	1	1.080 00	0.925 93	1.000 0	0.925 9	0.925 9	1
$i^{(2)}$	0.078 461	2	1.166 40	0.857 34	2.080 0	1.783 3	2.640 6	2
$i^{(4)}$	0.077 706	4	1.360 49	0.735 03	3.246 4	2.577 1	5.022 1	3
$i^{(12)}$	0.077 208	5	1.469 33	0.680 58	4.506 1	3.312 1	7.962 2	4
		6	1.586 87	0.630 17	5.866 6	3.992 7	11.365 1	5
δ	0.076 961	7	1.713 82	0.583 49	7.335 9	4.622 9	15.146 2	6
$(1+i)^{1/2}$	1.039 230	8	1.850 93	0.540 27	8.922 8	5.206 4	19.230 6	7
$(1+i)^{1/4}$	1.019 427	9	1.999 00	0.500 25	10.636 6	5.746 6	23.552 7	8
$(1+i)^{1/12}$	1.006 434	10	2.158 92	0.463 19	12.487 6	6.246 9	28.055 0	9
v	0.925 926	11	2.331 64	0.428 88	14.486 6	6.710 1	32.686 9	10
$v^{1/2}$	0.962 250	12	2.518 17	0.397 11	16.645 5	7.139 0	37.404 6	11
$v^{1/4}$	0.980 944	13	2.719 62	0.367 70	18.977 1	7.536 1	42.170 0	12
$v^{1/12}$	0.993 607	14	2.937 19	0.340 46	21.495 3	7.903 8	46.950 1	13
d	0.074 074	15	3.172 17	0.315 24	24.214 9	8.244 2	51.716 5	14
$d^{(2)}$	0.075 499	16	3.425 94	0.291 89	27.152 1	8.559 5	56.445 1	15
$d^{(4)}$	0.076 225	17	3.700 02	0.270 27	30.324 3	8.851 4	61.115 4	16
$d^{(12)}$	0.076 715	18	3.996 02	0.250 25	33.750 2	9.121 6	65.710 0	17
$i/i^{(2)}$	1.019 615	19	4.315 70	0.231 71	37.450 2	9.371 9	70.214 4	18
$i/i^{(4)}$	1.029 519	20	4.660 96	0.214 55	41.446 3	9.603 6	74.617 0	19
$i/i^{(12)}$	1.036 157	21	5.033 83	0.198 66	45.762 0	9.818 1	78.907 9	20
i/δ	1.039 487	22	5.436 54	0.183 94	50.422 9	10.016 8	83.079 7	21
$i/d^{(2)}$	1.059 615	23	5.871 46	0.170 32	55.456 8	10.200 7	87.126 4	22
$i/d^{(4)}$	1.049 519	24	6.341 18	0.157 70	60.893 3	10.371 1	91.043 7	23
$i/d^{(12)}$	1.042 824	25	6.848 48	0.146 02	66.764 8	10.528 8	94.828 4	24
		26	7.396 35	0.135 20	73.105 9	10.674 8	98.478 9	25
		27	7.988 06	0.125 19	79.954 4	10.810 0	101.994 1	26
		28	8.627 11	0.115 91	87.350 8	10.935 2	105.374 2	27
		29	9.317 27	0.107 33	95.338 8	11.051 1	108.619 8	28
		30	10.062 66	0.099 38	103.965 9	11.158 4	111.732 3	29
		31	10.867 67	0.092 02	113.283 2	11.257 8	114.713 6	30
		32	11.737 08	0.085 20	123.345 9	11.349 8	117.566 1	31
		33	12.676 05	0.078 89	134.213 5	11.435 0	120.292 5	32
		34	13.690 13	0.073 05	145.950 6	11.513 9	122.895 8	33
		35	14.785 34	0.067 63	158.626 7	11.586 9	125.379 3	34
		36	15.968 17	0.062 62	172.316 8	11.654 6	127.746 6	35
		37	17.245 63	0.057 99	187.102 1	11.717 2	130.001 0	36
		38	18.625 28	0.053 69	203.070 3	11.775 2	132.146 5	37
		39	20.115 30	0.049 71	220.315 9	11.828 9	134.186 8	38
		40	21.724 52	0.046 03	238.941 2	11.878 6	136.125 6	39
		41	23.462 48	0.042 62	259.056 5	11.924 6	137.966 8	40
		42	25.339 48	0.039 46	280.781 0	11.967 2	139.714 3	41
		43	27.366 64	0.036 54	304.243 5	12.006 7	141.371 8	42
		44	29.555 97	0.033 83	329.583 0	12.043 2	142.943 0	43
		45	31.920 45	0.031 33	356.949 6	12.077 1	144.431 7	44
		46	34.474 09	0.029 01	386.505 6	12.108 4	145.841 5	45
		47	37.232 01	0.026 86	418.426 1	12.137 4	147.175 8	46
		48	40.210 57	0.024 87	452.900 2	12.164 3	148.438 2	47
		49	43.427 42	0.023 03	490.132 2	12.189 1	149.631 9	48
		50	46.901 61	0.021 32	530.342 7	12.212 2	150.760 2	49
		60	101.257 06	0.009 88	573.770 2	12.233 5	151.826 3	50
		70	218.606 41	0.004 57	1 253.213 3	12.376 6	159.676 6	60
		80	471.954 83	0.002 12	2 720.080 1	12.442 8	163.975 4	70
		90	1 018.915 09	0.000 98	5 886.935 4	12.473 5	166.273 6	80
		100	2 199.761 26	0.000 45	12 723.938 6	12.487 7	167.480 3	90
					27 484.515 7	12.494 3	168.105 0	100

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Compound Interest

n	$(1+i)^n$	v^n	s_n	σ_n	$(Ia)_n$	$(Da)_n$	n	12%
1	1.120 00	0.892 86	1.000 0	0.892 9	0.892 9	0.892 9	1	i 0.120 000
2	1.254 40	0.797 19	2.120 0	1.690 1	2.487 2	2.582 9	2	$i^{(2)}$ 0.116 601
3	1.404 93	0.711 78	3.374 4	2.401 8	4.622 6	4.984 7	3	$i^{(4)}$ 0.114 949
4	1.573 32	0.635 52	4.779 3	3.037 3	7.164 7	8.022 1	4	$i^{(12)}$ 0.113 866
5	1.762 34	0.567 43	6.352 8	3.604 8	10.001 8	11.626 9	5	δ 0.113 329
6	1.973 82	0.506 63	8.115 2	4.111 4	13.041 6	15.738 3	6	$(1+i)^{1/2}$ 1.058 301
7	2.210 68	0.452 35	10.089 0	4.563 8	16.208 0	20.302 0	7	$(1+i)^{1/4}$ 1.028 737
8	2.475 96	0.403 88	12.299 7	4.967 6	19.439 1	25.269 7	8	$(1+i)^{1/12}$ 1.009 489
9	2.773 08	0.360 61	14.775 7	5.328 2	22.684 6	30.597 9	9	v 0.892 857
10	3.105 85	0.321 97	17.548 7	5.650 2	25.904 3	36.248 1	10	$v^{1/2}$ 0.944 911
11	3.478 55	0.287 48	20.654 6	5.937 7	29.066 5	42.185 8	11	$v^{1/4}$ 0.972 065
12	3.895 98	0.256 68	24.133 1	6.194 4	32.146 7	48.380 2	12	$v^{1/12}$ 0.990 600
13	4.363 49	0.229 17	28.029 1	6.423 5	35.125 9	54.803 8	13	d 0.107 143
14	4.887 11	0.204 62	32.392 6	6.628 2	37.990 6	61.431 9	14	$d^{(2)}$ 0.110 178
15	5.473 57	0.182 70	37.279 7	6.810 9	40.731 0	68.242 8	15	$d^{(4)}$ 0.111 738
16	6.130 39	0.163 12	42.753 3	6.974 0	43.341 0	75.216 8	16	$d^{(12)}$ 0.112 795
17	6.866 04	0.145 64	48.883 7	7.119 6	45.816 9	82.336 4	17	$i/i^{(2)}$ 1.029 150
18	7.689 97	0.130 04	55.749 7	7.249 7	48.157 6	89.586 1	18	$i/i^{(4)}$ 1.043 938
19	8.612 76	0.116 11	63.439 7	7.365 8	50.363 7	96.951 9	19	$i/i^{(12)}$ 1.053 875
20	9.646 29	0.103 67	72.052 4	7.469 4	52.437 0	104.421 3	20	i/δ 1.058 867
21	10.803 85	0.092 56	81.698 7	7.562 0	54.380 8	111.983 3	21	$i/d^{(2)}$ 1.089 150
22	12.100 31	0.082 64	92.502 6	7.644 6	56.198 9	119.628 0	22	$i/d^{(4)}$ 1.073 938
23	13.552 35	0.073 79	104.602 9	7.718 4	57.896 0	127.346 4	23	$i/d^{(12)}$ 1.063 875
24	15.178 63	0.065 88	118.155 2	7.784 3	59.477 2	135.130 7	24	
25	17.000 06	0.058 82	133.333 9	7.843 1	60.947 8	142.973 8	25	
26	19.040 07	0.052 52	150.333 9	7.895 7	62.313 3	150.869 5	26	
27	21.324 88	0.046 89	169.374 0	7.942 6	63.579 4	158.812 1	27	
28	23.883 87	0.041 87	190.698 9	7.984 4	64.751 8	166.796 5	28	
29	26.749 93	0.037 38	214.582 8	8.021 8	65.835 9	174.818 3	29	
30	29.959 92	0.033 38	241.332 7	8.055 2	66.837 2	182.873 5	30	
31	33.555 11	0.029 80	271.292 6	8.085 0	67.761 1	190.958 5	31	
32	37.581 73	0.026 61	304.847 7	8.111 6	68.612 6	199.070 0	32	
33	42.091 53	0.023 76	342.429 4	8.135 4	69.396 6	207.205 4	33	
34	47.142 52	0.021 21	384.521 0	8.156 6	70.117 8	215.362 0	34	
35	52.799 62	0.018 94	431.663 5	8.175 5	70.780 7	223.537 5	35	
36	59.135 57	0.016 91	484.463 1	8.192 4	71.389 4	231.729 9	36	
37	66.231 84	0.015 10	543.598 7	8.207 5	71.948 1	239.937 4	37	
38	74.179 66	0.013 48	609.830 5	8.221 0	72.460 4	248.158 4	38	
39	83.081 22	0.012 04	684.010 2	8.233 0	72.929 8	256.391 4	39	
40	93.050 97	0.010 75	767.091 4	8.243 8	73.359 6	264.635 2	40	
41	104.217 09	0.009 60	860.142 4	8.253 4	73.753 1	272.888 6	41	
42	116.723 14	0.008 57	964.359 5	8.261 9	74.112 9	281.150 5	42	
43	130.729 91	0.007 65	1 081.082 6	8.269 6	74.441 8	289.420 1	43	
44	146.417 50	0.006 83	1 211.812 5	8.276 4	74.742 3	297.696 5	44	
45	163.987 60	0.006 10	1 358.230 0	8.282 5	75.016 7	305.979 0	45	
46	183.666 12	0.005 44	1 522.217 6	8.288 0	75.267 2	314.267 0	46	
47	205.706 05	0.004 86	1 705.883 8	8.292 8	75.495 7	322.559 8	47	
48	230.390 78	0.004 34	1 911.589 8	8.297 2	75.704 0	330.857 0	48	
49	258.037 67	0.003 88	2 141.980 6	8.301 0	75.893 9	339.158 0	49	
50	289.002 19	0.003 46	2 400.018 2	8.304 5	76.066 9	347.462 5	50	
60	797.596 93	0.001 11	7 471.641 1	8.324 0	77.134 1	430.632 9	60	
70	2 787.799 83	0.000 36	23 223.331 9	8.330 3	77.540 6	513.913 8	70	
80	8 658.483 10	0.000 12	72 145.692 5	8.332 4	77.691 8	597.230 2	80	
90	26 891.934 22	0.000 04	224 091.118 5	8.333 0	77.747 0	680.558 1	90	
100	83 522.265 73	0.000 01	696 010.547 7	8.333 2	77.766 9	763.889 7	100	