

Actuarial Society of South Africa

Examiners' Report

May 2024

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

The examiners mark with candidates' answers where a question has several parts which follow on each other. It is therefore essential that candidates work consistently with their answers.

Many candidates can increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

Question 1

i.

Column A		Column B	
a	Annuity-certain	8	A series of regular payments for a fixed term.
b	Endowment assurance	4	A lump sum benefit on survival to the end of a specified term, or on death during the term.
c	Credit card	7	A flexible borrowing arrangement, with full or partial monthly repayments.
d	Zero-coupon bond	1	A fixed lump sum at a specified future date and no other positive cashflows.
e	Pure endowment	6	A lump sum benefit on survival to the end of a specified term.
f	Contingent annuity	5	A series of regular payments that are dependent upon certain events.

[4 marks total]

ii.

- Model development requires a considerable investment of time and expertise.
- The financial costs of developing a model and validating its results can be significant.
- Poor data quality will lead to model results that lack credibility.
- Model output may be difficult to interpret, especially in absolute terms.
- The results of complicated models can be difficult to communicate to the target audience.
- Model results are more useful for comparing results of input variations than for optimizing outputs.
- Models can look impressive when run on a computer, but this often leads to a false sense of security, despite flawed imitation of real-world systems (model did not pass validation and verification).
- Model users must fully comprehend the limitations of the model being used and the appropriateness of its results in the context to which it is applied.
- Several independent runs of stochastic models are needed to produce a range of possible outcomes.
- Models cannot incorporate all possible future events and are a necessary simplification of future expectations.

[Total 10]

This question was well answered in general.

A common error in ii. was to write down a general list of factors instead of focusing on the drawbacks of interpreting model output and communicating the results.

Question 2

$$PV(\text{Home Loan}) = 16,585 \ddot{a}_{12.5|3.5\%}^{(12)}$$

$$i = 0.03531$$

$$d^{(12)} = 0.034647 \quad i^{(12)} = 0.0347474$$

$$\ddot{a}_{12.5|3.5\%}^{(12)} = \frac{1-v^{12.5}}{d^{(12)}} = \frac{1-1.03531^{-12.5}}{0.034647} = 10.156782$$

$$\therefore PV(\text{Home Loan}) = 168,450.2335$$

$$PV(\text{Study Loan}) = 50,000 a_{19.5\%}^{(4)}$$

$$i = 0.099247$$

$$i^{(4)} = 0.095754$$

$$a_{19.5\%}^{(4)} = \frac{1-v}{i^{(4)}} = \frac{1-1.099247584^{-1}}{0.093515} = 0.942903$$

$$\therefore PV(\text{Study Loan}) = 47,145.1688$$

$$PV(\text{Personal Loan}) = 60,000 a_{0.5|16\%}^{(12)}$$

$$i^{(12)} = 0.149342$$

$$a_{0.5|16\%}^{(12)} = \frac{1-v}{i^{(12)}} = \frac{1-1.16^{-1}}{0.147506} = 0.478924$$

$$\therefore PV(\text{Personal Loan}) = 28,735.4425$$

$$\text{Hence } PV(\text{Total Outstanding Loans}) = 244,330.8449$$

$$\text{but } PV(\text{Total Outstanding Loans}) = 25,000 \bar{a}_{n|7.5\%}$$

$$\delta = \log(1.075) = 0.72320662$$

$$\bar{a}_{n|7.5\%} = \frac{1-1.075^{-n}}{\delta} = \frac{1-0.930233^n}{0.72320662}$$

$$\Rightarrow n = \frac{\log\left(1-0.72320662 \times \frac{244,330.8449}{25,000}\right)}{\log(0.930233)}$$

$$= 16.96504497 \text{ years}$$

[Total 14]

A common error was to use the incorrect interest rate with the different annuities. Attention to detail was required when writing out the final answer as the appropriate units (in this case years) must be added.

*The final annuity is a continuous annuity so the final answer should be given with decimals (**not** as an integer value).*

Question 3

i.

The variation of interest rates by term.

[1]

ii.

$$y_8 = \frac{0.05}{(1 - 0.01 \times 8)^3} = 0.06421$$

$$Price = 100(1 + y_8)^{-8} = 60.78$$

[2]

iii.

$$y_{12} = \frac{0.05}{(1 - 0.01 \times 12)^3} = 0.07337$$

$$y_4 = \frac{0.05}{(1 - 0.01 \times 4)^3} = 0.05651$$

$$(1 + f_{4,8})^8 = \frac{(1 + y_{12})^{12}}{(1 + y_4)^4} = 1.87713$$

$$\Rightarrow f_{4,8} = 0.08190$$

$$Price = 100(1 + f_{4,8})^{-8} = 53.27$$

[4]

[Total 7]

A common error in iii. was not knowing that $f_{4,8}$ is the interest rate needed to calculate the price for the specified zero-coupon bond purchased in four years' time.

Question 4

i.

$$PV(\text{Liabilities}) = 100,000v^{10}$$

$$PV(\text{Assets}) = Xv^8 + Yv^{12}$$

$$\text{First condition: } 100,000v^{10} = Xv^8 + Yv^{12}$$

$$DMT(\text{Liabilities}) = 10$$

$$DMT(\text{Assets}) = \frac{8Xv^8 + 12Yv^{12}}{Xv^8 + Yv^{12}} = 10$$

$$\text{Second condition: } 8Xv^8 + 12Yv^{12} = 10(Xv^8 + Yv^{12})$$

$$12Yv^{12} - 10Yv^{12} = 10Xv^8 - 8Xv^8 \Rightarrow 2Yv^{12} = 2Xv^8$$

$$\Rightarrow \frac{X}{Y} = v^4 \Rightarrow \frac{0.64Y}{Y} = v^4 \Rightarrow 0.64 = v^4$$

$$\text{Therefore } i = 0.64^{-1/4} - 1 = 11.8034\%$$

[10]

ii.

$$\text{Convexity (Liabilities)} = \frac{10 \times 11 \times 100,000v^{12}}{100,000v^{10}} = \frac{2,823,426.022399}{32,197.323659} = 87.6913$$

$$\begin{aligned} \text{Convexity(Assets)} &= \frac{8 \times 9 \times 40,000v^{10} + 12 \times 13 \times 62,500v^{14}}{40,000v^8 + 62,500v^{12}} = \frac{927,282.92138, 995,043.17301}{16,155.32912 + 16,042.19331} \\ &= \frac{2,922,326.09439}{32,197.52243} = 90.7625 \end{aligned}$$

Hence Convexity (Liabilities) < Convexity (Assets)

∴ Immunisation has been achieved.

[5]

iii.

- A flat yield curve is assumed at all terms, which is unlikely to be the case in practice.
- The cost of immunisation may be excessive, as continuous rebalancing will be needed.
- The required zero-coupon bonds may not be available.
- There may be uncertainty in the size and timing of the assets and liabilities so that the cashflows will only be approximate.
- Immunisation theory relies on “small” changes in the interest rates.

- Immunisation reduces the probability of profits.
- Immunisation ignores expenses, which may change over time.

[max 3 marks]

[Total 18]

In i. most candidates could set up the two equations that needed to be solved to calculate the interest rate. Fewer candidates could solve the two equations to give the correct numerical answer for the interest rate.

In ii. a common error was to calculate the second derivative (which is correct) but to label it as convexity. Calculating either the convexity or the second derivative would be acceptable if labelled correctly.

Question 5

i.

$$\text{Repayment} = \frac{250,000}{12 \times a_{\overline{5}|}^{(12)} \times v^3}$$

where

$$i = \left(1 + \frac{i^{(2)}}{2}\right)^2 - 1 = 13.4225\%$$

$$i^{(12)} = 12.661389\%$$

$$a_{\overline{5}|}^{(12)} = 3.6906$$

$$\Rightarrow \text{Repayment} = R8,236.88$$

[4]

ii.

$$\text{Loan outstanding} = 8,236.88 \times 12 \times a_{\overline{2}|}^{(12)} = R173,836.614725$$

$$\text{New repayment} = \frac{173,836.61 - 80,000}{a_{\overline{9}|@ \frac{i^{(12)}}{12}}} = \frac{93,836.61472536}{a_{\overline{9}|@ \frac{i^{(12)}}{12}}}$$

where

$$\frac{i^{(12)}}{12} = 0.5654\%$$

$$i^{(12)} = 6.78497\%$$

$$a_{\overline{9}|@ \frac{i^{(12)}}{12}} = 8.75075022$$

New repayment = R10,723.27

[6]

iii.

$$\begin{aligned} \text{Net present value (NPV)} &= 12 \times 10,000 a_{\overline{1.5}|@5\%}^{(12)} \\ &= 120,000 \times 1.4435 \\ &= 173,218.50 \\ &< 173,836.61 \\ &= \text{Loan outstanding} \end{aligned}$$

Since the NPV at an annual percentage rate of return (APR) of 5% is less than the outstanding balance, we would need to discount at a lower interest rate for the NPV to equal the outstanding balance.

Hence $\text{APR} < 5\%$.

[4]

[Total 14]

A common error in ii. was to use the original interest rate when calculating the new repayment.

There are several ways of doing iii. If the APR is calculated numerically, it must be calculated with linear interpolation to obtain full credit.

Question 6

i.

Test for a capital gain: $i = 8\% \Rightarrow i^{(4)} = 0.0777$

$$g(1 - t_1) = 0.1(1 - 0.25) = 0.075$$

Since $i^{(4)} > g(1 - t_1)$ there is a capital gain, and the bond should be assumed to be redeemed at the latest possible date.

Calculate the original purchase price P at 8%:

$$\begin{aligned} P &= 10,000 \times 0.75 \times a_{\overline{10}|}^{(4)} + 100,000v^{10} - (100,000 - P) \times 0.3v^{10} \\ \Rightarrow P &= \frac{7,500 \times 6.9082 + 70,000 \times 0.4632}{(1 - 0.3 \times 0.4632)} \\ \Rightarrow P &= 97,828.82 \end{aligned}$$

[8]

ii.

Test for a capital gain: $i = 6\% \Rightarrow i^{(4)} = 0.0587$

Since $i^{(4)} < g(1 - t_1)$ there is no capital gain, and the bond should be assumed to be redeemed at the earliest possible date.

Calculate the purchase price P' at 6%:

$$\begin{aligned} P' &= 10,000 \times 0.75 \times a_{\overline{3}|}^{(4)} + 100,000v^3 \\ \Rightarrow P' &= 7,500 \times 2.7324 + 100,000 \times 0.8396 \\ \Rightarrow P' &= 104,455.11 \end{aligned}$$

[4]

ii.

Yes. Linda sold the bond at a higher price than she purchased it for and thus made a capital gain.

[2]

[Total 14]

A common error in ii. was to assume the same capital gains tax position as in i. applies, although the investor's details have changed.

A common error in iii. was to use previous information about interest rates to determine whether a capital gain has been made. This type of test was no longer valid as the fixed-interest bond had been sold before redemption.

Overall question was well answered.

Question 7

i.

The nominal interest rate is $1.04 \times 1.077 - 1 = 12.008\% \approx 12\%$

$$Price(B) = 100,000 \times 46 = 4,600,000$$

$$Price(A) = 500,000a_{\overline{10}|} + 80,000(Ia)_{\overline{5}|} + v^5[400,000a_{\overline{5}|} + 0.1Y(Ia)_{\overline{5}|}] = 4,600,000$$

$$\text{where } a_{\overline{10}|} = \frac{1-1.12008^{-10}}{0.12008} = 5.648373201$$

$$a_{\overline{5}|} = \frac{1-1.12008^{-5}}{0.12008} = 3.604061903$$

$$\ddot{a}_{\overline{5}|} = \frac{1-1.12008^{-5}}{0.12008/1.12008} = 4.036837656$$

$$(Ia)_{\overline{5}|} = \frac{\ddot{a}_{\overline{5}|} - 5v^5}{i} = 9.999304$$

$$\Rightarrow Y = \frac{4,600,000 - 441,855.44}{1.12008^{-5} \times 0.1 \times 9.999304} = R278,823.71$$

[13]

ii.

$$\begin{aligned} 46 &= \frac{3 \times 1.02}{1.04(1+i)} + \frac{3 \times 1.02 \times 1.04}{1.04^2(1+i)^2} + \frac{3 \times 1.02 \times 1.04^2}{1.04^3(1+i)^3} + \dots \\ &= \frac{3 \times 1.02}{1.04} \left(\frac{1}{1+i} + \frac{1}{(1+i)^2} + \frac{1}{(1+i)^3} + \dots \right) \\ &= \frac{3 \times 1.02}{1.04} \times a_{\infty|} = \frac{3 \times 1.02}{1.04} \times \frac{1}{i} \\ &\Rightarrow i = 6.396321\% \end{aligned}$$

[7]

iii.

No.

The NPV is a function of the interest rate used to calculate it. When the NPV of two projects are plotted against the interest rate, there can be a cross over point where $NPV(C) > NPV(D)$ switches to $NPV(C) < NPV(D)$, or vice versa.

The internal rate of return (IRR) for C and D will be unique if they exist and if $IRR(C) > IRR(D)$, this will always be the case.

[3]

Common errors in i. were

- *Using numerically incorrect interest rate*
- *Not using nominal interest rate*
- *Incorrect timeline of payments*

Common errors in ii. were

- *Growth factors incorrectly applied to dividends*
- *Not dividing interest rate into inflation and real yield*