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SOUTH AFRICAN ANNUITANT STANDARD MORTALITY TABLES 1996–2000 (SAIML98 and SAIFL98)

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ABSTRACT
This paper describes the data and the processes used to produce the first standard tables of mortality of South African immediate annuitants. A parametric curve was fitted to the data from the normal retirement ages up to age 85. Below the normal retirement ages the rates increasingly reflected the impact of higher mortality due to ill-health retirements and so the curve was blended into that of the most recent standard table of life-assured mortality (SA85–90). Above age 85 the estimates were thought to be unreliable and the extrapolation of the curve fit to the younger ages did not allow for the expected fall in the rate of increase in the rates with age. Thus rates above this age were estimated using a relationship proposed by Coale and Kisker. It was not possible to produce select rates or to decide on a trend in these rates over time.

KEYWORDS
South Africa; annuitant; mortality; standard table

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1. INTRODUCTION

Although the Actuarial Society of South Africa has produced a number of standard tables of the mortality of lives assured (SA56–62, SA72–77 and SA85–90), this is the first time a standard set of mortality rates that reflect the experience of South African immediate annuitants has been produced.

2. BACKGROUND

2.1 This investigation can be dated back to 1994 when Rob Thomson, then a member of the Continuous Statistical Investigations (CSI) Committee of the Actuarial Society of South Africa (ASSA) sent out initial requests to life offices and others who might have significant annuitant and pensioner mortality experience.

2.2 Initially the Committee hoped to investigate pensioner mortality as well, and to produce estimates of mortality rates within a year or two, but offices were very slow to submit data. By April 1999 only four life offices had submitted data: Commercial Union and Old Mutual (for 1995 only); Momentum (for 1995 and 1996); Sanlam (for 1995–1997). Apart from difficulties in getting sufficient data, investigations of the data highlighted several problems, in particular implausibly low rates for those still within the guarantee period. So the Committee decided to concentrate efforts on investigating annuitant mortality, and life offices who had submitted problematic data were asked to remedy the faults that were found with their data.

3. DATA

3.1 DESCRIPTION OF THE DATA INCLUDED IN THE INVESTIGATION

3.1.1 The data are in terms of numbers of lives and distinguish between the following classes of lives:

- voluntary and compulsory purchases of immediate annuities by individuals;
- men and women;
- annuities within the guarantee period and those beyond this period;
- amount of annuity (at year end) in six bands:
  - less than R1000;
  - R1000 to R2999–99;
  - R3000 to R9 999–99;
  - R10 000 to R29 999–99;
  - R30 000 to R99 999–99; and
  - R100 000 and greater.

3.1.2 The data captured in each category were:

- age last birthday;
- central exposure to risk;
- amount of annuities exposed to risk; and
- amount of annuities in respect of deaths.

3.2 DATA EXCLUDED FROM THE INVESTIGATION

The following data were excluded from the investigation:

- back-to-back annuity arrangements;
- deferred, reversionary and contingent annuities;
- annuities issued in a territory outside the Republic of South Africa;
- annuities payable in a currency other than the South African rands;

– annuities paid to or on behalf of a pension or provident fund; and
– annuities with missing data (date of birth, sex, whether voluntary or compulsory, whether annuity payments commenced or ceased during the year).

3.3 THE AMOUNT OF EXPERIENCE

Data, covering the period 1996 to 2000 were collected from six companies: Fedsure; Liberty Life; Momentum; Old Mutual; Sanlam and Southern Life. One company submitted data for only 1997, 1999 and 2000 and another for only 1999 and 2000. The years of exposure and number of deaths for lives and amounts are shown in total in Table 1 and for lives aged below 60 and aged 60 and above in Table 2:

Table 1. Exposure and deaths by number of lives and amount of payment

	Lives		Amount (millions)	
	male	female	male	female
Deaths	32 107	9 129	184	47
E_x^c	967 450	566 388	6 657	2 971

Table 2. Exposure and deaths by number of lives below age 60 and aged 60 and above

	Male		Female	
	<60	60+	<60	60+
Deaths	2 270	29 828	956	8 173
	150 968	816 482	173 196	393 191
% of E_x^c	16%	84%	31%	69%

3.4 EXPERIENCE BY LIVES

3.4.1 By way of comparison Table 3 shows the data available to the Continuous Mortality Investigation Bureau (CMIB) of the Faculty and Institute of Actuaries in the UK to produce their standard annuitant mortality tables (Continuous Mortality Investigation Bureau, 1999: 27–44). Clearly, in total this investigation has considerably more years of exposure than that used by the CMIB. It is also of interest to note how the market in immediate annuities has shrunk in the UK over the past 70 years.

3.4.2 Table 4 shows the age ranges for which there are sufficient data to facilitate the production of statistically reliable rates. Comparing the data available for this investigation with those available to the CMIB we see that the age range over which there might be considered to be sufficient data is much younger in South Africa. However, many of these data are of limited usefulness since the experience below age 60 becomes increasingly influenced by ill-health retirements, the younger the age being considered. The shortage of data above age 95 means that mortality rates at the advanced ages will have to be produced by extrapolation.

Table 3. UK CMIB Immediate-annuitants mortality experience

	Male		Female	
	1921–25	1991–94	1921–25	1991–94
Duration 0:				
deaths	123	104	205	146
E_x^c	4 001	2 934	9 635	4 033
Duration 1+:				
deaths	3 350	2 886	7 738	5 863
E_x^c	49 122	35 690	153 625	66 470

Table 4. Ages with significant data

	Total	Exposure >= 100	Deaths >= 10
SA:			
male	0–103	20–94	40–96
female	3–110	21–95	41–97
UK (ult):			
male	22–107	61–96	67–100
female	10–108	60–101	70–104

3.5 AVERAGE SIZE OF ANNUITIES

Table 5 shows the average size of annuities for the two periods 1996–1998 and 1999–2000 and over the whole period. Although the increase in the annuities over time seems consistent with inflation over the same period, the sizes of the annuities are remarkably low. In part this may be due to tax limits placed on the size of retirement annuities used to eventually purchase annuities.

Table 5. Average amount per life year (rands per annum)

	1996–2000	1996–1998	1998–2000
Male	6 881	6 260	7 603
Female	5 252	4 733	5 834
All	6 279	5 702	6 942

3.6 PROBLEMS WITH THE DATA

3.6.1 Inspection of the data led to the identification of a number of problems which are discussed below.

3.6.2 Data submitted by two of the companies contained negative deaths and years of exposure. Further investigation revealed that the negative deaths arose from contra entries of deaths incorrectly recorded in the previous year. Thus these negative deaths need to remain in the investigation. This does introduce a slight inaccuracy in that we are effectively assuming that the number of negative deaths in the first year of the investigation is the same as the number of deaths in the final year that would need to be removed, but the numbers involved are not significant.

3.6.3 Similarly, negative exposures were used to correct the estimate of the years of exposure where it turned out that a death occurred before the year in question. These were also included in the investigation, again on the assumption that the negative number of years of exposure in the first year of the investigation was of roughly the same absolute magnitude as the excess exposure incorrectly recorded in the final year of the investigation.

3.6.4 The second problem with the data is that data from four of the offices had been rounded down. Whereas at the younger ages this does not make any noticeable difference to the rates of mortality, at the very old ages it does introduce an upward bias in the rates estimated. For this and various other reasons it might be considered appropriate not to use the estimates for extremely old ages.

3.6.5 A third problem arose when close inspection of some individual records at old ages for one of the life offices showed some examples where the age categorisation was not consistent from one year to the next. Since this sort of error can only be spotted by tracing the record of individual lives from one year to the next, it is impossible to discover from aggregated data and hence to know how extensive this error was. The office in question was asked by the CSI Committee to investigate but at the time of writing was still to respond. It has been assumed that this error was not material.

3.6.6 Finally, as will be seen from the presentation of the observed mortality rates calculated from these data, the rates of mortality derived from the data of two of the life offices were implausibly low.

4. RESULTS

4.1 COMPARISON WITH STANDARD TABLES

Figures 1 and 2 show a comparison of the 95% confidence intervals around the force of mortality, $\mu_{x+\frac{1}{2}}$, with the force of mortality derived from various standard tables. From these we see that for men there is a reasonable degree of consistency of rates at the higher ages down to ages in the high 60s, while for women this consistency extends down to the ages in the low 60s. Below these ages the observed rates are increasingly higher than those derived from the standard tables, which can undoubtedly be attributed to the impact of early ill-health retirements at these ages. For men of ages above 65 the rates are best approximated by a(90) ultimate less one year and PA(90) ultimate less three years. The rates for women of ages above 60 appear to be equally well approximated by all three tables (a(90) ultimate less one year, PA(90) ultimate less two years and PFL80 ultimate less two years). The differences below these ages are due to the fact that the standard tables have been graduated to produce rates ignoring ill-health retirements.

Figure 1. Confidence intervals around the observed rates vs. various standard tables: male

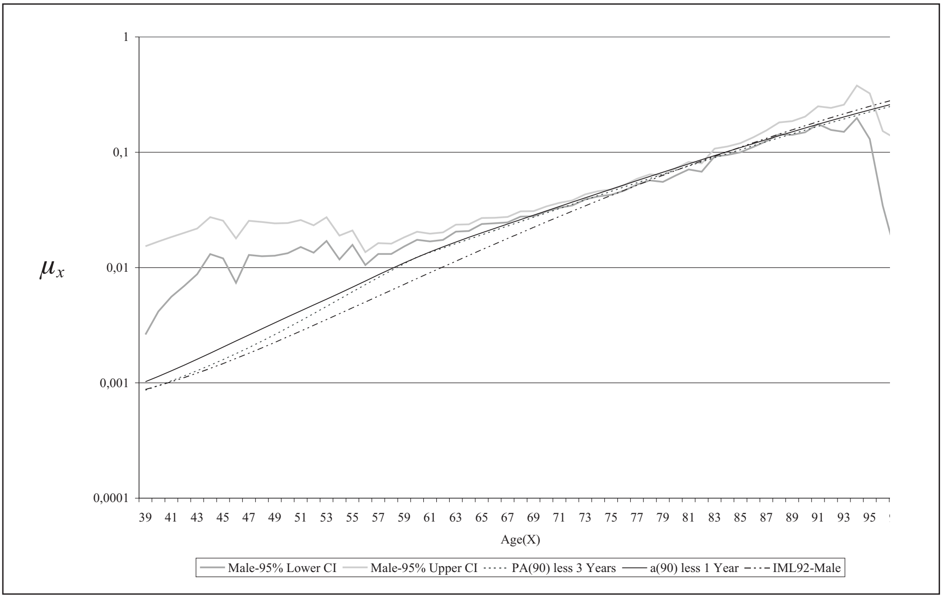


Figure 2. Confidence intervals around the observed rates vs. various standard tables: female

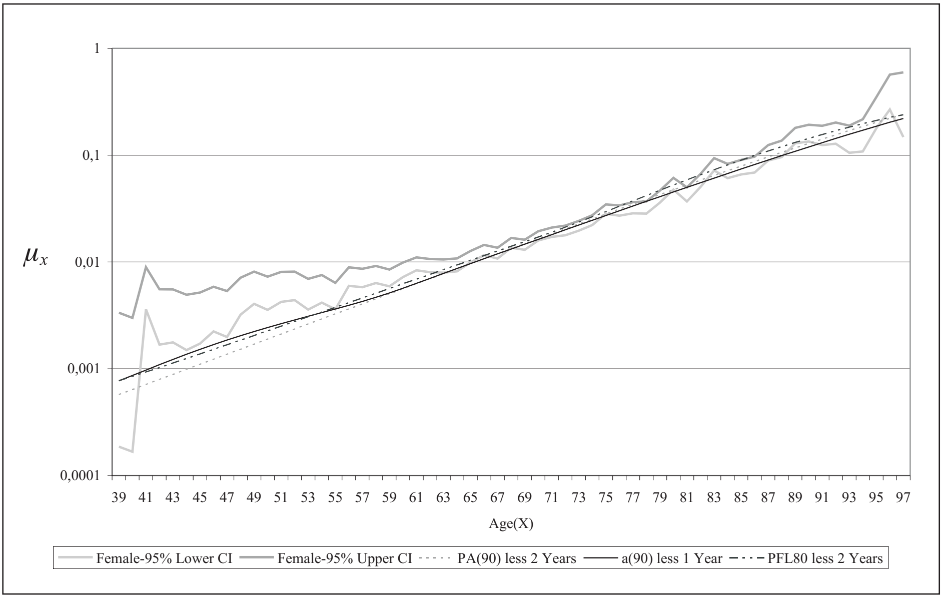


Figure 3. Rates for amounts and rates for lives vs. various standards: male

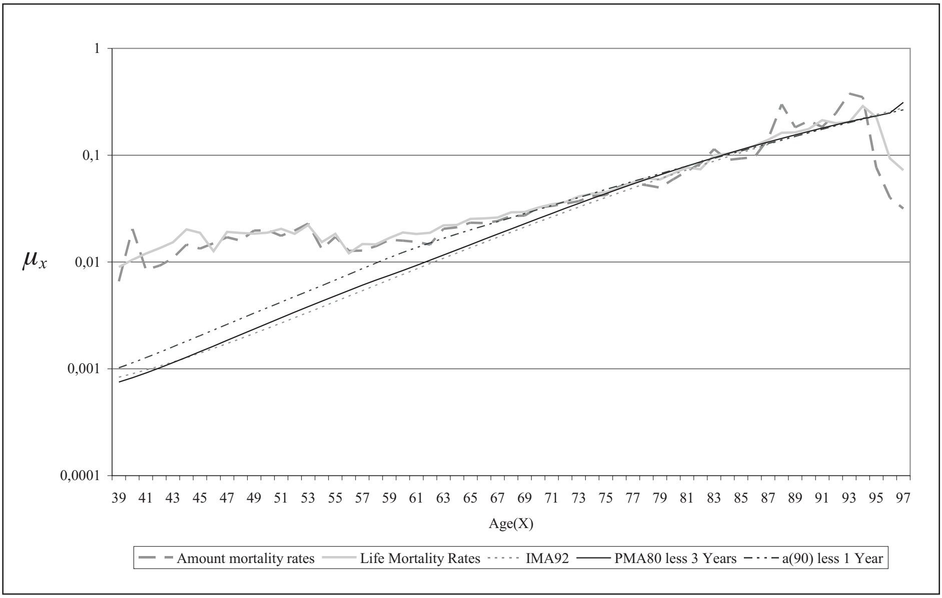
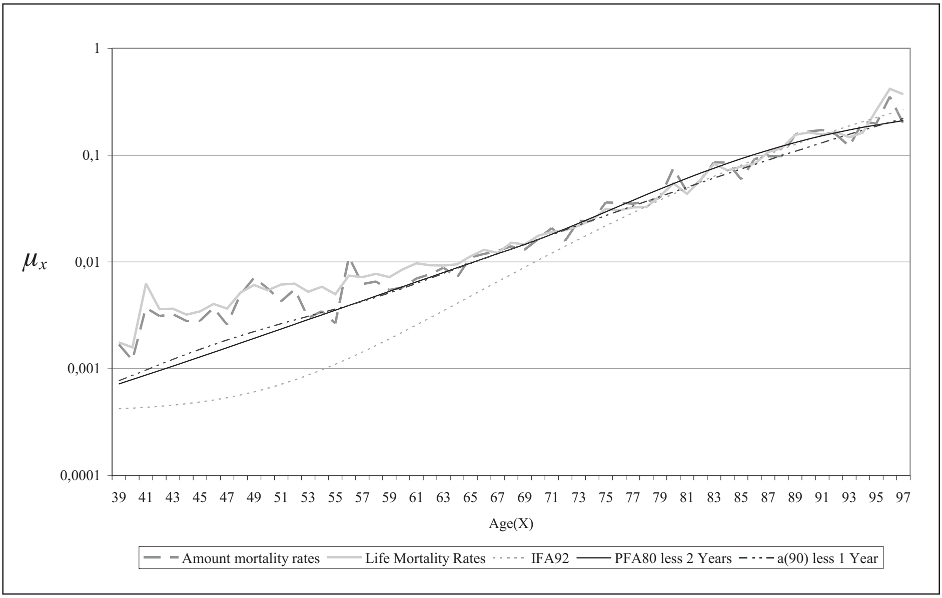


Figure 4. Rates for amounts and rates for lives vs. various standards: female



4.2 IMPACT OF SIZE OF ANNUITY

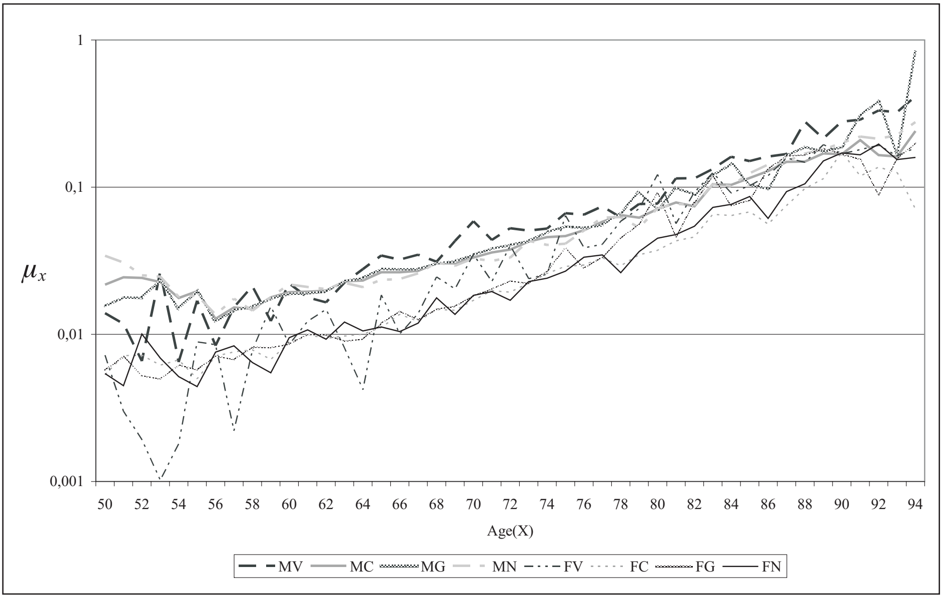
Figures 3 and 4 compare the rates calculated on the basis of the number of years of exposure lived (lives) with those based on the exposure weighted by the amount of the annuities (amount). From these we see that there is no evidence to suggest that the one set of rates is significantly different from the other. Thus it is hardly surprising that attempts to approximate the rates using standard tables based on amounts do not produce better fits to the rates based on amount of annuity than standard tables based on lives.

4.3 MORTALITY RATES BY CLASS OF ANNUITY

As can be seen from the comparison of the rates of mortality for the various categories of annuity (i.e. voluntary, compulsory, in guarantee period, beyond guarantee period) in Figure 5 and the standardised mortality ratios (SMRs) in Table 6 what evidence there is of differences between classes of annuity shows differences in a direction opposite to what would be expected, rates of mortality for voluntary annuitants being higher than those for compulsory annuitants and the rates of those within the guarantee period being higher than those beyond this period. This, together with the fact that there are insufficient data for either voluntary annuitants or those still within the guarantee period, suggests it could be spurious to produce standard tables for anything other than all classes combined. In Figure 5, the following codes are used:

– F: female;
– M: male;

Figure 5. Mortality rates by class of annuity: female (lower curves) and male (upper curves)



– C: compulsory purchase;
– V: voluntary purchase;
– G: within the guarantee period;
– N: not in the guarantee period.

Table 6. Observed mortality compared with expected on the basis of the rates for all classes combined

	Sex	Compulsory	Voluntary	Within guarantee period	Beyond guarantee period
Directly standardised mortality rate	male	0,035	0,043	0,037	0,034
	female	0,017	0,024	0,019	0,017
Standardised mortality ratio	male	0,983	1,228	1,029	0,955
	female	0,933	1,298	1,049	0,934

4.4 DIFFERENCES BETWEEN LIFE OFFICES

4.4.1 As can be seen from Figure 6, the mortality rates for four of the life offices are essentially the same, but two of the offices experienced substantially lower mortality rates. Comparison of the 95% confidence interval around the rates for these two companies combined with the most recent experience from the UK (Figure 7) show that rates from these companies are implausibly low. Enquiries of the individual life offices failed to establish why the rates were lower.

4.4.2 As the two life offices with low mortality contributed only a small proportion of the total experience, their exclusion does not result in significantly different mortality rates, as shown in Figure 8.

4.4.3 Although not shown here, similar patterns were observed for women.

5. GRADUATION AND EXTENSION

5.1 GRADUATION

5.1.1 After exploring various alternatives, the authors decided to graduate the observed mortality rates using two methods. The first, was to fit a GM(r,s) equation using generalised linear models (McCullagh & Nelder, 1989) and maximum-likelihood estimation. The GM (r,s) equation is defined by:

$$GM_x(r,s) = \sum_{i=1}^{r} \alpha_i x^{i-1} + \exp\left\{ \sum_{i=r+1}^{r+s} \alpha_i x^{i-r-1} \right\}$$

where: $\mu_x = GM_x(r,s)$,

and a_i is a constant. This was converted to Chebyshev polynomials for convenience of order of magnitude and consistency of parameters as the order of the polynomials are changed. This family of models was chosen for the following reasons: they were flexible

Figure 6. Mortality rates by life office: male

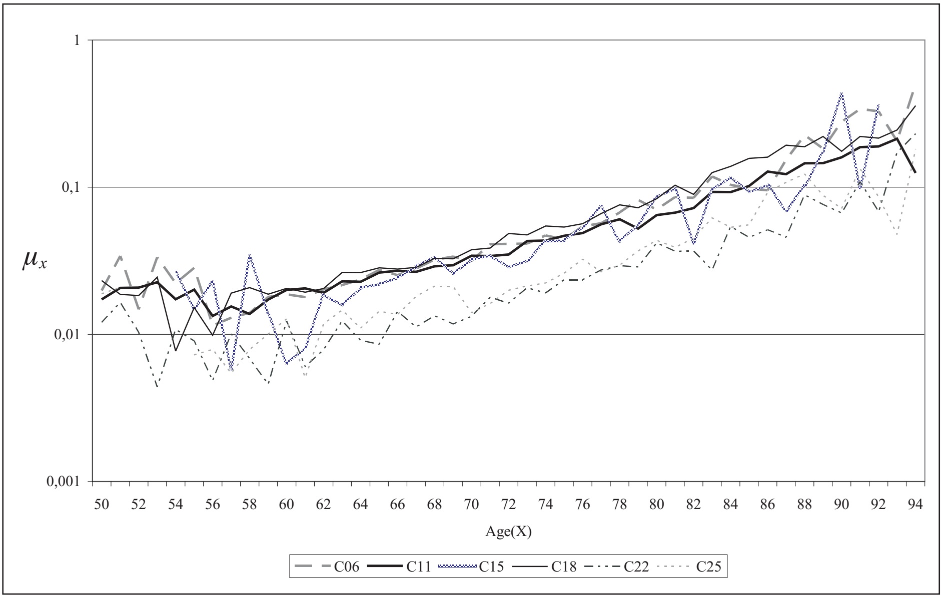
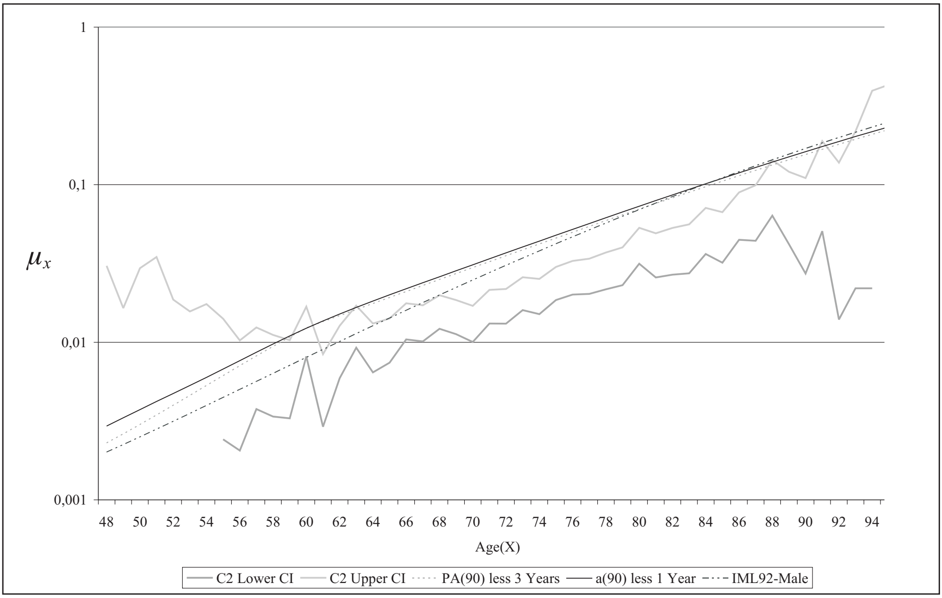


Figure 7. Confidence intervals around rates for outlier life offices (C2) vs. UK rates: male



enough to facilitate consideration of a wide range of alternative parametric equations; they were used to graduate most of the recent UK life tables (cf. e.g. Continuous Mortality Investigation Bureau, 1999); and there were sufficient data to fit over a wide range of ages. The second method, chosen on the grounds that the observed rates were very close to rates derived from the a(90) standard table, was to graduate rates by reference to this standard table using:

$$\mu_x = a + b\mu^s_{x-1};$$

where μ^s_x is the force of mortality from the a(90) ultimate table. In this case the rates were fitted using weighted least squares. Fitting was done using Genstat v6.1.

5.1.2 For ease of computation, following Renshaw (1995) (and others, e.g. Continuous Mortality Investigation Bureau (1976: 61) before him), the ages were scaled so that:

$$y = \frac{x-70}{50}.$$

Initially the curves were fitted using data for the ages 39 to 94 for men and 42 to 95 for women. These ages were chosen as being the ages for which there were at least five expected deaths and at least 100 years of exposure, which is the guideline used by the CMIB (1999).

5.1.3 The results of these graduations are presented, together with the critical value at the 5% level of significance, in Tables 7 and 8 and shown graphically in Figures 9 and 10.

Figure 8. Rates with and without (C1) outliers: male

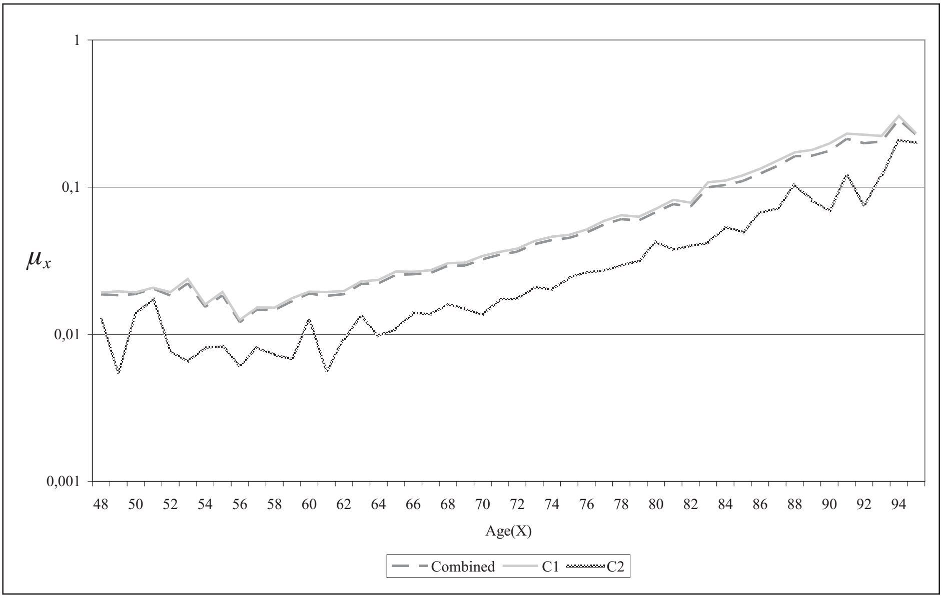


Table 7. Chi-squared values vs. critical value at the 5% level of significance (in brackets) for various graduations over the whole range: male

	GM(0,3)	GM(0,4)	GM(1,2)	GM(1,3)	GM(2,2)	GM(2,3)	GM(2,4)	a + b* [a(90)-1yr]
Chi-squared Value	148,65	135,32	145,9	144,17	137,74	132,2	120,49	343,05
	(70,99)	(69,83)	(70,99)	(69,83)	(69,83)	(68,67)	(67,65)	(70,99)

Table 8. Chi-squared values vs. 5% critical value (in brackets) for various graduations over the whole range: female

	GM(0,3)	GM(0,4)	GM(1,2)	GM(1,3)	GM(2,2)	GM(2,3)	GM(2,4)	a + b* [a(90)-1yr]
Chi-squared Value	92,23	86,8	83,99	83,98	81,79	71,52	70,68	133,37
	(68,67)	(67,65)	(68,67)	(67,65)	(67,65)	(66,34)	(65,17)	(68,67)

5.1.4 Not surprisingly, given the large number of lives involved as well as the distortion introduced by the ill-health retirements at the younger ages, all the graduations are statistically significantly different from the observed rates over the whole range of ages. Since, in the main, any standard table produced would be used for calculations applied to those who purchased the annuities at or above normal retirement age, the goodness of fit of the various models over the age ranges 65 to 92 for men and 60 to 88 for women was also considered. (Although normal retirement can occur from age 55, and inspection of the exposure shows a marked increase from this age, it is likely that these earlier retirements would still include a number of ill-health retirees.) These results are presented in Table 9.

Table 9. Chi-squared values vs. 5% critical value (in brackets) for various graduations over selected age ranges

Age Range	GM(0,2)	GM(0,3)	GM(1,2)	GM(1,3)	GM(2,2)	GM(2,3)	a + b* [a(90)-1yr]
men 65–92	118,06	36,32	35,43	35,17	35,43	35,01	77,46
	(38,89)	(37,65)	(37,65)	(36,42)	(36,42)	(35,17)	(36,42)
women 60–88	92,08	54,24	51,83	50,55	50,96	50,53	118,15
	(40,11)	(38,89)	(38,89)	(37,65)	(37,65)	(36,42)	(60,48)

5.1.5 Clearly the $GM(r,s)$ method produces better fits than graduation using a standard table. As might be expected, the fits with the lowest chi-squared statistic are those with the most parameters, namely, $GM(2,3)$ for both men and women. However, if the criterion for selecting the best graduation is that each additional parameter reduce the

Figure 9. Observed vs. graduated rates: male

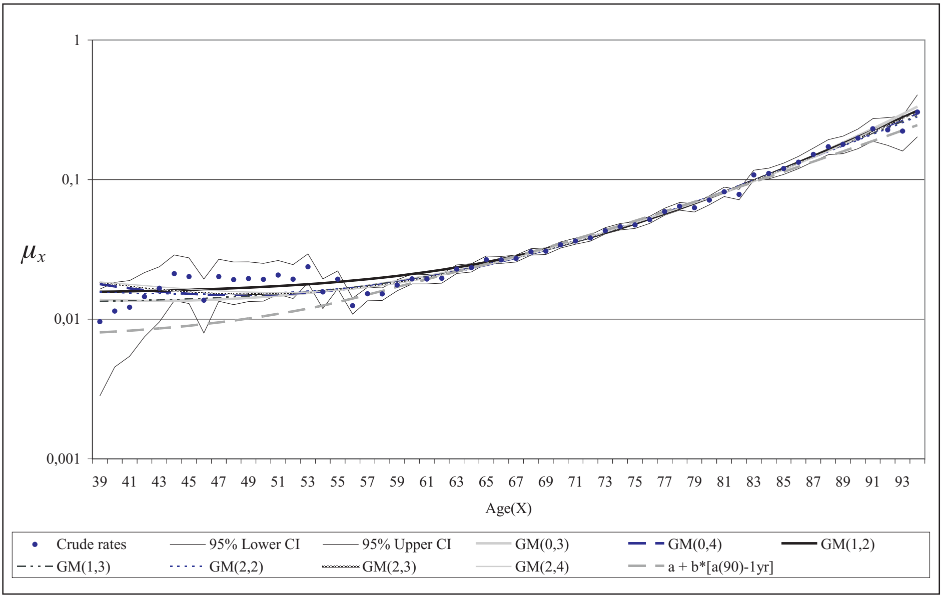
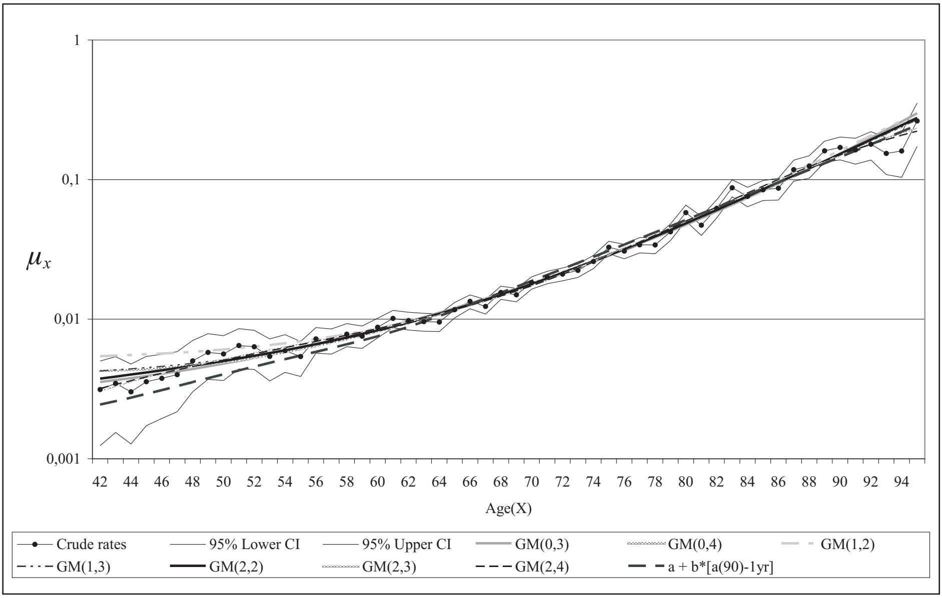


Figure 10. Observed vs. graduated rates: female



chi-squared value by 4 (roughly implying a significant difference at the 5% level) the curves produced by *GM*(1,2) are selected. The fits were subjected to further tests (signs, serial correlations and standardised deviations—see Appendix B) and found to be acceptable in all cases, except that for both men and women there were two observations more than two standard deviations below the graduated rates when we would expect less than one, and in the case of women three observations more than two standard deviations above the graduated rates.

5.1.6 It was thus decided to accept these graduations, namely:

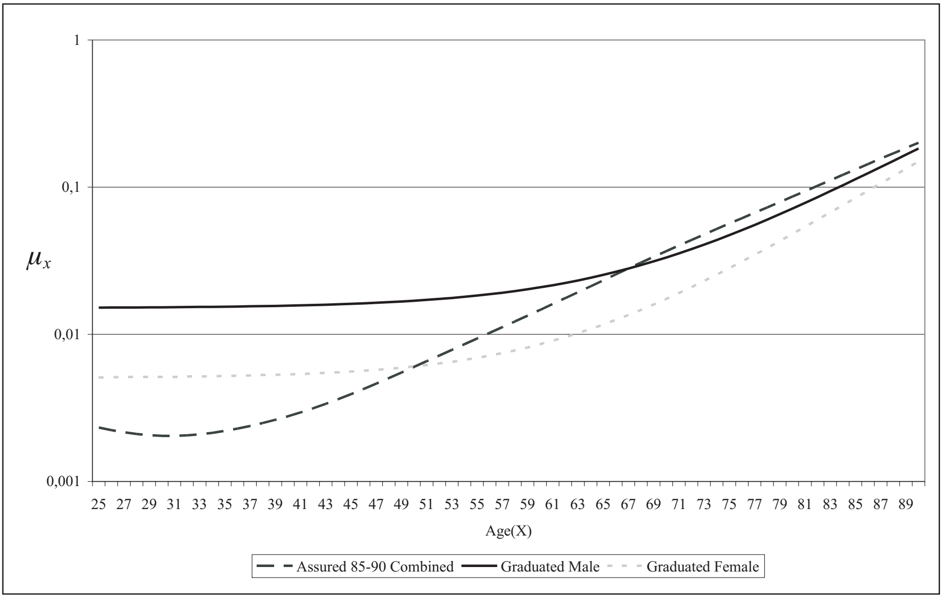
$$\mu_{x+½} = 0{,}015\ 203\ 11 + \exp\left(-3{,}983\ 763\ 63 + 5{,}765\ 160\ 46\frac{x-70}{50}\right) \text{ for men;}$$

and

$$\mu_{x+½} = 0{,}005\ 073\ 33 + \exp\left(-4{,}383\ 786\ 22 + 6{,}321\ 845\ 561\frac{x-70}{50}\right) \text{ for women.}$$

5.1.7 The graduated rates are compared with the SA85–90 ultimate rates for lives assured in Figure 11. From this we note that above age 65 the relationship (or at least the ranking) between the rates is as expected, the rates for assured lives being higher than those for annuitants. Below age 65, however, the assured mortality rates are significantly lower than the rates for male annuitants, even falling below the rates for female annuitants below age 45. This gives a measure of the effect of ill-health retirements and suggests

Figure 11. Graduated vs. SA85–90



that, at least for men, it would be better to use the SA85–90 rates than the extrapolation of the graduation curves below age 65. (The force of mortality underlying the SA85–90 rates was approximated by:

$$\mu_{x-\frac{1}{2}} = -\ln(1-q_x);$$

and the rates blended using cubic splines set to reproduce the rates and first derivative of the rates for SA85–90 at age 61 and for the GM(1, 2) curve at age 71.) In the case of women, following the suggestion by Dorrington & Rosenberg (1996), the rates below 69 (chosen as the age which best allowed the two sets of rates to blend together) were taken as 45% of the SA85–90 ultimate rates.

5.2 EXTENDING THE TABLES TO AGE 100

5.2.1 Comparison of the curves with the data suggests that empirical rates become increasingly unreliable after age 92. (The reason for this is not clear but probably has something to do with misstatement of age.) However, it would be necessary for a standard table to extend to at least age 100. One option would be to simply extrapolate the fitted curve to the older ages. Other options would be to extrapolate on the basis of the relationship between the fitted rates and those of another standard table at the advanced ages, or to extrapolate on the basis of the pattern of the relationship between rates at advanced ages to those in the 80s such as that estimated by Coale & Guang (1989), which was used in the production of the WHO life tables (Murray, Ferguson, Lopez et al, 2003), or Coale & Kisker (1990).

5.2.2 Coale & Guang (op. cit.) found that it is suitable to assume that $\ln({}_5m_x/{}_5m_{x-5})$ declines linearly with increasing age for ages over 80. Thus, if one is able to establish an estimate of mortality at a suitably advanced age (${}_5m_{105}={}_5m_{75}$ +0,66 was used by the WHO) then one is able to derive rates at ages above 85.

5.2.3 Working with data for individual ages, Coale & Kisker (op. cit.) developed this work further, namely that:

$$m_x = m_{84}\exp\left[\sum_{y=85}^{x} k_{85} + (y-85)s\right] \text{for } x > 84;$$

where:

$$k_{85}=\ln(m_{85})–\ln(m_{84});$$

and s was estimated so that m_{110} was 1 for men and 0,8 for women. In the interests of robustness they suggested that an average of the mortality rates for ages 82 to 86 be used in place of m_{84} and that k_{85} and s be estimated as follows:

$$k_{85} = \frac{1}{7}\ln\left(\frac{m_{88}}{m_{81}}\right); \text{ and}$$

$$s = \frac{1}{325}\left\{\ln\left(\frac{m_{84}}{m_{110}}\right) + 26k_{85}\right\};$$

Figure 12. Comparison of alternative fits to old ages: female

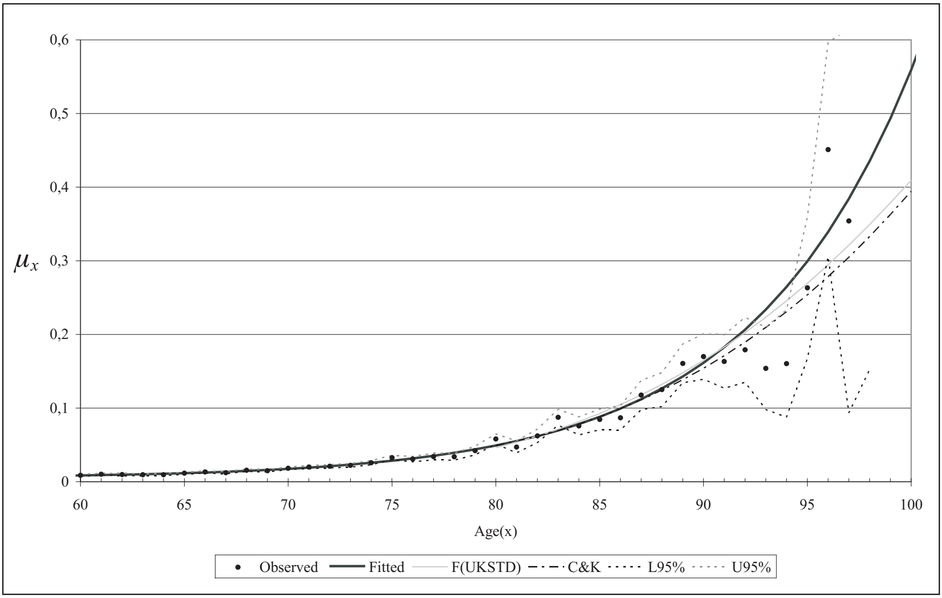
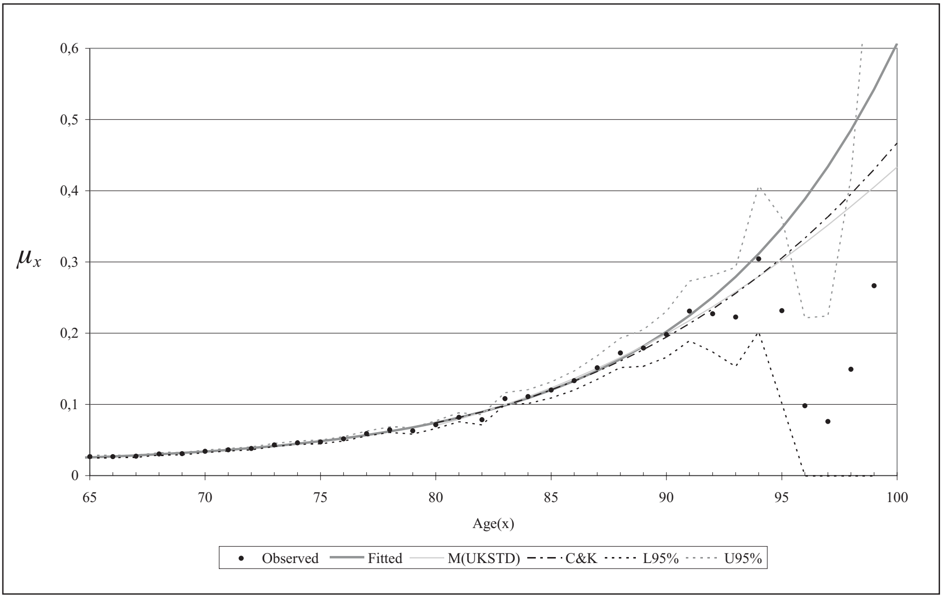


Figure 13. Comparison of alternative fits to old ages: male



and this suggestion has been followed here. Wilmoth (1995) suggests that if the estimates of mortality rates for ages 85 to 100 are considered to be free of bias then one can avoid having to fix m_{110} by fitting a quadratic curve to the observed rates using weighted least squares. The pattern of observed mortality rates at these high ages does not suggest that they are reliable.

5.2.4 Examining data from Sweden from 1895 and Japan from 1950, Wilmoth (op. cit.) found that while there is some evidence of modest annual decline in m_{110} for women to the level of about 0,8 (or lower) over these periods, there was no evidence of any consistent trend in m_{110} for men.

5.2.5 Figures 12 and 13 compare the estimates derived using these three methods. In the second method IML92Base and IFLp2Base were used as standards with coefficients of a linear relationship determined from the fit of the observed rates to the standard over the age range 80 to 90. In the third method it was assumed that the relationships found to hold for m_x also apply to $\mu_{x+\frac{1}{2}}$. From these comparisons it is clear that extrapolation of the graduation fitted to the observed rates for lower ages produces rates that are too high at the older ages. The graduation to the standard table and the method proposed by Coale & Kisker (op. cit.) produce very similar answers. Since the Coale–Kisker method is more general (not being dependent on any pattern forced on the rates by the choice of standard) it was decided to use these rates in producing the standard table extending the age range to 110.

6. SELECT RATES

6.1 No data were collected on duration of policies and hence select rates could not be produced. The most recent study of the mortality of immediate annuitants in the UK (Continuous Mortality Investigation Bureau, 1999) found a ratio of $\mu_{[x]} / \mu_x$ of 73,8001% for men and 71,724% for women, but there is no reason to suppose that such relationships would apply to South African immediate annuitants.

6.2 The standard table of rates, labelled, following the CMIB convention but prefacing with SA for South Africa, SAIML98 and SAIFL98 are given in Appendix A.

7. TIME TREND

7.1 Figures 14 and 15 show the mortality rates for individual years and Table 10 the SMRs using the rates for all years as standard.

Table 10. Standardised mortality ratio by year using all years combined as standard

Sex	1996–1998	1999–2000	1996	1997	1998	1999	2000
male	1,038	0,948	1,042	1,066	1,006	0,957	0,940
female	1,072	0,913	1,134	1,037	1,052	0,942	0,886

Figure 14. Trend over time: male

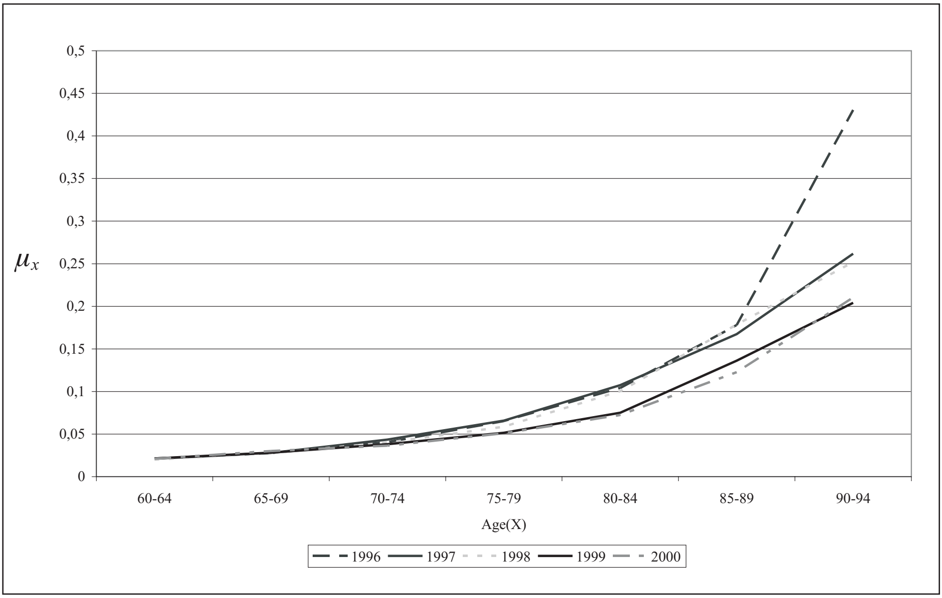
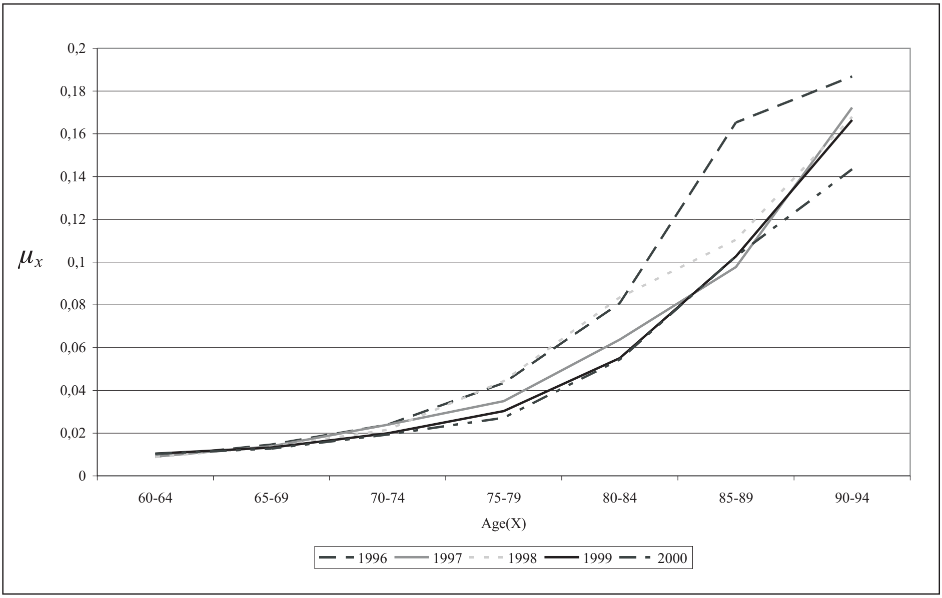


Figure 15. Trend over time: female



7.2 Curves fitted to SMRs of the rates for ages 50 to 94 over time gives a good fit (R^2 over 80% for men and 90% for women), however, there are a number of difficulties with using these result to project future mortality:
– These fits are heavily influenced by lower rates in the most recent two years, which could be underestimating mortality because of deaths not yet processed.
– CMIB uses long periods (between investigations) to determine the trend (not a short period of investigation).
– The implied decreases of around 3% a year for men and nearly 6% a year for women are much higher than those observed in the UK.
– Comparison of population mortality rates produced by Dorrington, Moultrie & Timæus (2004) with the South African Life Tables 1984–86 (Central Statistical Service, 1987) and those produced by Dorrington (1998) suggest that there has been little decrease in population mortality of those over 65 between 1984–86 and 1996–2001 (even within the population groups).
– The pattern of decline found in the UK suggested a greater rate of decline at younger ages falling to zero by age 110. This is not the pattern displayed by the above data.

7.3 The authors have recommended to the Actuarial Society of South Africa that the standard does not include a method for projecting rates; a note should, they suggest, be included, explaining that there are too few data from which to decide a trend, and suggesting that those that need to, apply the rule applied in the UK for many years, namely, a reduction of one year of age for every twenty years projected. The CSI Committee intends to collect data annually and each year the data will be examined to see if a clear trend in the rates can be identified to facilitate the determination of a reliable basis for the projection of rates of South African annuitant mortality.

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APPENDIX A
SAIFL98 AND SAIML98 STANDARD TABLES

Age	Female		Male	
	$\mu_{x+½}$	q_x	$\mu_{x+½}$	q_x
40	0,00129	0,00129	0,00286	0,00286
41	0,00137	0,00137	0,00304	0,00304
42	0,00146	0,00146	0,00326	0,00325
43	0,00158	0,00158	0,00351	0,00350
44	0,00170	0,00170	0,00378	0,00377
45	0,00184	0,00184	0,00409	0,00408
46	0,00200	0,00200	0,00444	0,00443
47	0,00217	0,00217	0,00483	0,00482
48	0,00237	0,00237	0,00527	0,00526
49	0,00259	0,00259	0,00576	0,00574
50	0,00283	0,00283	0,00630	0,00628
51	0,00310	0,00309	0,00688	0,00686
52	0,00339	0,00338	0,00753	0,00750
53	0,00371	0,00370	0,00823	0,00820
54	0,00405	0,00404	0,00900	0,00896
55	0,00443	0,00442	0,00984	0,00979
56	0,00484	0,00483	0,01076	0,01070
57	0,00530	0,00528	0,01177	0,01170
58	0,00580	0,00578	0,01288	0,01280
59	0,00635	0,00633	0,01411	0,01401
60	0,00697	0,00694	0,01548	0,01536
61	0,00764	0,00761	0,01698	0,01684
62	0,00839	0,00835	0,01858	0,01841
63	0,00922	0,00917	0,02023	0,02002
64	0,01012	0,01007	0,02192	0,02168
65	0,01112	0,01105	0,02368	0,02340
66	0,01220	0,01213	0,02551	0,02518
67	0,01339	0,01330	0,02742	0,02704
68	0,01469	0,01458	0,02942	0,02899
69	0,01607	0,01594	0,03153	0,03104
70	0,01755	0,01740	0,03375	0,03319
71	0,01923	0,01905	0,03609	0,03545
72	0,02114	0,02092	0,03865	0,03791
73	0,02331	0,02304	0,04151	0,04066

74	0,02576	0,02544	0,04473	0,04374
75	0,02855	0,02815	0,04834	0,04719
76	0,03172	0,03122	0,05238	0,05104
77	0,03531	0,03469	0,05693	0,05534
78	0,03938	0,03862	0,06203	0,06014
79	0,04401	0,04305	0,06775	0,06551
80	0,04926	0,04806	0,07417	0,07149
81	0,05521	0,05371	0,08138	0,07816
82	0,06197	0,06009	0,08947	0,08558
83	0,06964	0,06727	0,09854	0,09384
84	0,07834	0,07535	0,10873	0,10303
85	0,08820	0,08442	0,12013	0,11319
86	0,09906	0,09431	0,13257	0,12416
87	0,11101	0,10507	0,14613	0,13596
88	0,12410	0,11671	0,16090	0,14862
89	0,13842	0,12927	0,17695	0,16218
90	0,15403	0,14276	0,19438	0,17666
91	0,17100	0,15718	0,21329	0,19208
92	0,18940	0,17255	0,23376	0,20845
93	0,20930	0,18884	0,25590	0,22578
94	0,23074	0,20605	0,27983	0,24408
95	0,25378	0,22414	0,30563	0,26334
96	0,27848	0,24306	0,33343	0,28354
97	0,30487	0,26278	0,36335	0,30466
98	0,33298	0,28321	0,39549	0,32665
99	0,36283	0,30430	0,42999	0,34948
100	0,39445	0,32595	0,46695	0,37309
101	0,42781	0,34807	0,50651	0,39740
102	0,46292	0,37056	0,54879	0,42235
103	0,49975	0,39332	0,59392	0,44784
104	0,53825	0,41623	0,64202	0,47377
105	0,57836	0,43918	0,69322	0,50004
106	0,62001	0,46206	0,74764	0,52652
107	0,66312	0,48476	0,80541	0,55310
108	0,70757	0,50716	0,86665	0,57964
109	0,75325	0,52916	0,93148	0,60603
110	0,80000	0,55067	1,00000	0,63212

APPENDIX B
TEST STATISTICS FOR GM(r,s)

$$\mu_{x+½} = a + by + \exp(c + dy + ey^2) \text{ where } y = \frac{x-70}{50}$$

	FEMALE						
		GM(0,2)	GM(0,3)	GM(1,2)	GM(1,3)	GM(2,2)	GM(2,3)
a				0,00507	-0,02248	0,00053	-0,01686
b						-0,01424	0,02676
c		-3,98631	-4,03290	-4,38379	-3,21949	-4,07994	-3,36897
d		4,87875	4,39819	6,32185	2,06364	5,62375	1,65059
e			3,19848		4,36484		5,91846
Chi-squared		92,08	54,24	51,83	50,55	50,96	50,53
Signs test		0,19	-0,56	-0,19	-0,93	-0,93	-0,93
Serial correlations		1,46	-0,74	-0,97	-1,15	-1,07	-1,19
Standard deviations test	Expected	Actual					
	0,04	0	0	0	0	0	0
	0,62	2	0	2	2	2	1
	3,94	8	6	4	4	4	5
	9,90	4	10	9	11	11	11
	9,90	6	9	9	6	6	6
	3,94	4	1	2	3	3	3
	0,62	3	2	2	2	2	2
	0,04	2	1	1	1	1	1
	Chi-squared	119,68	30,66	30,88	31,64	31,64	29,09

MALE							
		GM(0,2)	GM(0,3)	GM(1,2)	GM(1,3)	GM(2,2)	GM(2,3)
a				0,01520	-0,02583	0,01617	-0,02373
b						0,00381	0,05546
c		-3,37599	-3,38202	-3,98376	-2,82046	-4,03540	-2,85759
d		4,00653	3,23075	5,76516	1,81871	5,86780	0,94919
e			3,18819		3,86084		5,59569
Chi-squared		118,06	36,32	35,43	35,17	35,45	35,01
Sign test		0,00	0,38	0,76	0,76	0,76	0,76
Serial correlations		2,74	-1,21	-1,16	-1,17	-1,19	-1,16
Standard deviations test	Expected	Actual					
	0,04	2	0	1	1	1	0
	0,60	2	1	1	1	1	2
	3,81	2	4	3	3	3	4
	9,56	8	8	7	7	7	6
	9,56	1	10	10	10	10	12
	3,81	7	4	5	5	5	3
	0,60	5	1	1	1	1	1
	0,04	1	0	0	0	0	0
	Chi-squared	173,41	0,91	26,32	26,32	26,32	5,75