

Actuarial Society of South Africa

Examiners Report

May 2025

Subject A211

General comments

Please note that different answers may be obtained to those shown in these solutions depending on whether intermediary figures were obtained from tables or from calculators, but candidates were not penalised for this.

Also, note that there are often alternative ways to reach the same final solution so that the solutions in this report should not be seen as the only solutions available.

The examiners mark with the candidates' answers where a question has several parts which follow on each other. It is therefore essential that candidates work consistently with their answers.

Many candidates can also increase their probability of passing this exam by practising good exam technique, which is often missing in its most basic form.

QUESTION 1

a.

$$\frac{i^{(2)}}{2} = 0.0255$$

$$\left(1 + \frac{i^{(2)}}{2}\right)^2 = \exp(\delta) \Rightarrow (1 + 0.0255)^2 = \exp(\delta)$$

$$1.05165025 = \exp(\delta) \Rightarrow \delta = 0.050360597 = 5.0361\%$$

b.

$$\left(1 + \frac{i^{(0.2)}}{0.2}\right)^{0.2} = (1 + i) = \exp(\delta) = \left(1 + \frac{i^{(2)}}{2}\right)^2 = 1.05165025$$

$$\Rightarrow i^{(0.2)} = 0.057268517 = 5.7269\%$$

c.

$$\left(1 - \frac{d^{(6)}}{6}\right)^6 = (1 + 0.0255)^{-2} = 0.95088648$$

$$\Rightarrow \frac{d^{(6)}}{6} = 0.008358306 = 0.8358\%$$

d.

$$v^{10} = \exp(-\delta \times 10) = \exp(-0.050360597 \times 10) = 0.604347467$$

$$1 - d = v = 0.950886476 \Rightarrow d = 0.049113524$$

$$(I\ddot{a})_{\overline{10}|} = \frac{\ddot{a}_{\overline{10}|} - 10v^{10}}{d} = \frac{8.055877477 - 10 \times 0.604347467}{0.049113524} = 40.9745145$$

Question was well done in general.

If parts (a), (b) and (c) were not rounded appropriately, full credit was not awarded.

QUESTION 2

i.

$$79(1+y_4)^4 = 100 \Rightarrow y_4 = \left(\frac{100}{79}\right)^{\frac{1}{4}} - 1 = 0.060701608$$

$$(1+y_4)^4 = (1+y_2)^2(1+f_{2,2})^2$$

$$y_2 = 0.071512286$$

$$92 = 3(1+y_1)^{-1} + 3(1+y_2)^{-2} + 103(1+y_3)^{-3}$$

$$\begin{aligned} y_1 &= \left(\frac{92 - 3(1+y_2)^{-2} - 103(1+y_3)^{-3}}{3} \right)^{-1} - 1 \\ &= \left(\frac{92 - 3(1.071512286)^{-2} - 103(1.06)^{-3}}{3} \right)^{-1} - 1 \\ &= 0.032244266 \end{aligned}$$

ii.

$$f_{0,1} = y_1 = 0.032244266$$

$$(1+y_2)^2 = (1+f_{0,1})(1+f_{1,1})$$

$$f_{1,1} = \frac{(1+y_2)^2}{1+y_1} - 1 = \frac{(1.071512286)^2}{1.032244266} - 1 = 0.112274117$$

$$(1+y_3)^3 = (1+y_2)^2 \times (1+f_{2,1}) = (1+f_{0,1})(1+f_{1,1})(1+f_{2,1})$$

$$\begin{aligned} f_{2,1} &= \frac{(1+y_3)^3}{(1+y_2)^2} - 1 = \frac{(1+y_3)^3}{(1+f_{0,1})(1+f_{1,1})} - 1 \\ &= 0.03734516 \end{aligned}$$

$$(1+y_4)^4 = (1+y_3)^3 \times (1+f_{3,1}) \Rightarrow (1.060701608)^4 = (1.06)^3 \times (1+f_{3,1})$$

$$f_{3,1} = 0.0628$$

Several alternative solutions are available for this question.

QUESTION 3

i.

$$\begin{aligned} PV &= e^{-\int_0^t \delta(s) ds} \\ &= e^{-\int_0^t 0.02 ds} \\ &= e^{-[0.02s]_0^t} \\ &= e^{-0.02t} \end{aligned}$$

ii.

$$\begin{aligned} \bar{a}_{\bar{t}|} &= \int_0^t e^{-\int_0^u \delta(s) ds} du \\ &= \int_0^t e^{-\int_0^u 0.02 ds} du \\ &= \int_0^t e^{-0.02u} du \\ &= -\frac{1}{0.02} e^{-0.02u} \Big|_0^t \\ &= \frac{1 - e^{-0.02t}}{0.02} \end{aligned}$$

iii.

$$(\bar{I}\bar{a})_{\bar{t}|} = \left(\frac{\bar{a}_{\bar{t}|} - te^{-0.02 \times t}}{0.02} \right)$$

iv.

$$\begin{aligned} PV &= 100\bar{a}_{\overline{10}|} - 1 \times (\bar{I}\bar{a})_{\overline{10}|} \\ &= 100 \times \frac{1 - e^{-0.02 \times 10}}{0.02} - \left(\frac{\bar{a}_{\overline{10}|} - 10e^{-0.02 \times 10}}{0.02} \right) \\ &= 906.3462 - 43.8077 \\ &= R 862.5385 \end{aligned}$$

A common error was to not answer the question as it was asked.

For instance, the question asked to state an expression in terms of $\delta(t)$ and then simplify as far as possible. Often the numerical value of $\delta(t)$ would be substituted immediately. Full credit was not awarded as initial steps were missed out on.

Candidates are reminded to read questions carefully.

QUESTION 4

i.

$$PV(\text{Liabilities}) = 204,020 \times v^7 + 980,402 \times v^{11} = 357,820.45$$

$$\begin{aligned} DMT(\text{Liabilities}) &= \frac{7 \times 204,020 \times v^7 + 11 \times 980,402 \times v^{11}}{204,020 \times v^7 + 980,402 \times v^{11}} \\ &= 10.00000017 \\ &\approx 10 \text{ years} \end{aligned}$$

ii.

$$PV(\text{Assets}) = Xv^5 + Yv^{15}$$

$$\text{First condition} \Rightarrow Xv^5 + Yv^{15} = 357,820.45 \dots (1)$$

$$DMT(\text{Assets}) = \frac{5 \times X \times v^5 + 15 \times Y \times v^{15}}{Xv^5 + Yv^{15}}$$

$$\text{Second condition} \Rightarrow 5 \times X \times v^5 + 15 \times Y \times v^{15} = 3,578,205.525 \dots (2)$$

Solve

$$X = 357,820.45 \times (1+i)^5 - Yv^{10}$$

$$X = Yv^{10}$$

$$(1) = (2) \Rightarrow Y = \frac{357,820.45 \times (1+i)^5}{2}$$

$$\text{Hence } Y = R1,046,943 \text{ and } X = R322,402$$

- iii. Calculate the respective convexities of the assets and the liabilities

$$\text{as } c(i) = \frac{P''(i)}{P(i)}$$

where $P(i)$ denotes the present value of the cashflows at interest rate i and $''$ denotes the second derivative with respect to i .

If $c_{Liabilities}(i) < c_{Assets}(i)$, immunisation has been achieved.

A common error in part(i) was not adding the time unit in the final answer

Part (ii) was answered well.

In part (iii) the definition of any notation used was asked for, but this was often not given.

QUESTION 5

- i.

$$\begin{aligned} \text{Flat Rate} &= \frac{\text{Total Payment} - \text{Original Loan}}{\text{Original Loan} \times \text{Loan Term}} \\ &= \frac{P \sum_{k=1}^n k - L}{Ln} = \frac{P \frac{n(n+1)}{2} - L}{Ln} \end{aligned}$$

- ii.

$$\begin{aligned} I_5 &= L_{4+} i \\ &= (4Pa_{\overline{n-4}|} + P(Ia)_{\overline{n-4}|}) \times i \\ C_5 &= 5P - (4Pa_{\overline{n-4}|} + P(Ia)_{\overline{n-4}|}) \times i \end{aligned}$$

- iii.

$$\begin{aligned} L_{4+} &= 4Pa_{\overline{n-4}|} + P(Ia)_{\overline{n-4}|} = Xa_{\overline{n-4}|} \\ X &= P \left(4 + \frac{(Ia)_{\overline{n-4}|}}{a_{\overline{n-4}|}} \right) \end{aligned}$$

iv.

$$C_5 = P \left(4 + \frac{(Ia)_{\overline{n-4}|}}{a_{\overline{n-4}|}} \right) - (4Pa_{\overline{n-4}|} + P(Ia)_{\overline{n-4}|}) \times i$$

v.

The capital portion on the restructured loan is greater than on the original loan because $4 + \frac{(Ia)_{\overline{n-4}|}}{a_{\overline{n-4}|}} > 5$ when i is positive.

vi.

The total interest paid on the restructured loan will be less than on the original loan because capital is paid off faster.

Hence, the flat rate of interest on the restructured loan will be less than on the original loan.

Common errors in part (ii):

- *incorrect payment size for level annuity in calculating L_{4+}*
- *incorrect payment size to subtract the interest component from*

Parts (v) and (vi) required candidates to methodically separate out the aspects of the loan prior to the restructure and post the restructure.

A common error was to assume an increasing payment pattern is larger in size than a level payment pattern, which is not the case in the earlier stages of repayment.

QUESTION 6

i.

Test for a capital gain: $g(1 - t_1) = 0.1(1 - 0.3) = 0.07$

Since $i = 7.5\% > g(1 - t_1)$ there is a capital gain and the worst case for the investor is when the bond is redeemed as late as possible.

As the bond was purchased at the lowest possible price to ensure a net effective yield of 7.5%, we therefore have that $n = 6$. At any earlier n , the net yield will be greater.

ii.

Test for a capital gain: $g(1 - t_1) = 0.1(1 - 0.25) = 0.075$

Since $i = 7.5\% = g(1 - t_1)$ the investment will earn the same yield on any redemption date $4 \leq n \leq 6$.

iii

$$j = (1 + 0.075)(1 + 0.02) - 1 \\ = 9.65\%$$

The capital gains test from (i) remains unchanged and so $n = 6$.

$$P = 10,000 \times 0.1 \times (1 - 0.3) \times a_{\overline{6}|@j} \\ + 10,000v_j^6 \\ - (10,000 - P) \times 0.38 \times v_j^6$$

$$\Rightarrow v_j^6 = 0.575371$$

$$\text{and } a_{\overline{6}|@j} = 4.400298$$

$$\Rightarrow P = \frac{700 \times 4.400298 + 6,200 \times 0.575371}{1 - 0.38 \times 0.575371}$$

$$\Rightarrow P = R 8,507.63$$

Candidates are reminded to establish a logical flow in the argument when conducting a capital gains test.

The investor's required yield should be compared to the net coupon rate, in order to establish the worst-case scenario. This then informs the assumption for term (n) in the calculation to determine the maximum price or minimum yield. Easy partial marks can be lost for simply comparing the interest rates and saying $n=6$, without establishing the worst-case scenario for the investor.

Part (i) was well answered.

*The common error in part (ii) was not realizing that **any** time between 4 and 6 would give the appropriate yield.*

Common errors in part (iii) were

- not calculating the nominal yield correctly*
- not checking that the capital gain test from part (i) still holds.*

QUESTION 7

i.

$$\begin{aligned}PV_{\text{purchase \& construction outflow}} &= R175m + 50m \times v + R25m \times v^2 \\ &= R244.8546753m\end{aligned}$$

$$\begin{aligned}PV_{\text{maintenance}} &= v^4 \times R3m \times \bar{a}_{\infty|j_1} = R3m \times (1.055)^{-4} \times 104.9992063 \\ &= R254.2713522m\end{aligned}$$

$$\begin{aligned}j_1 &= \frac{1.055}{1.045} - 1 = 0.0095693 \dots \Rightarrow \delta = \ln(1 + j_1) = 0.00952388 \\ PV_{\text{sale inflow}} &= R50m \times a_{\overline{4}|0.055} = R50m \times 3.505150122 \\ &= R175.2575061m\end{aligned}$$

$$\begin{aligned}PV_{\text{rates}_1} &= v^4 \times (R9m \times \bar{a}_{\overline{6}|0.055} + R6m \times (I\bar{a})_{\overline{6}|0.055}) \\ &= R120.396965m\end{aligned}$$

$$\bar{a}_{\overline{6}|0.055} = 5.131681572$$

$$(I\bar{a})_{\overline{6}|0.055} = 17.16093232$$

$$\begin{aligned}PV_{\text{rates}_2} &= R45m \times v^{10} \times (1.035\bar{a}_{\overline{1}|0.055} + 1.035^2 v \bar{a}_{\overline{1}|0.055} + \dots) \\ &= R45m \times \bar{a}_{\overline{1}|0.055} \times v^{10} \times 1.035(1 + 1.035v + 1.035^2 v^2 + \dots) \\ &= R45m \times \bar{a}_{\overline{1}|0.055} \times v^{10} \times 1.035\ddot{a}_{\infty|j_2} \\ &= R1,400.478265m\end{aligned}$$

$$\bar{a}_{\overline{1}|0.055} = 0.973701058$$

$$\ddot{a}_{\infty|j_2} = 52.75$$

$$j_2 = \frac{1.055}{1.035} - 1 = 0.019323671$$

$$\begin{aligned}NPV &= R175.2575061m + R120.396965m + R1,400.478265m - R244.8546753m \\ &\quad - R254.2713522m \\ &= R1,197.006709m\end{aligned}$$

ii.

Show that NPV for cashflows by time 10 is positive.

$$\begin{aligned}PV_{outflow} &= PV_{purchase \& construction outflow} + PV_{main cap 6 years} \\&= R244.8546753m + v^4 \times R3m \times \bar{a}_{6|@j_1} \\&= R244.8546753m + 14.12255406m \\&= R258.9772294m\end{aligned}$$

$$\bar{a}_{6|@j_1} = 5.831789$$

$$\begin{aligned}PV_{inflow} &= PV_{sale inflow} + PV_{rates_1} \\&= R175.2575061m + R120.396965m \\&= R295.6544711m\end{aligned}$$

$PV_{inflow} > PV_{outflow} \therefore$ The discounted payback period is before time 10.

iii.

Nothing would happen.

The discounted payback period and hence payback period are only affected by cashflows before the break-even point, not after.

Candidates that work methodically by separating the different payment streams in part (i) typically scored well, but those that tried to take on too much at once often confused themselves.

A common error in part (i) was applying the growth factors to the annuities incorrectly.

In the $PV_{maintenance}$ the payments and growth is continuous leading to a continuous annuity with a new interest rate.

For PV_{rates_2} payments are also continuous, but increases are discrete (annually in arrear), which leads to a different type of simplified expression.

In part (iii), the common error was not realizing that changes in the cash flow after the DPP would not affect its value.