

EXAMINATION

6 May 2025 (am)

Subject A211 — Financial Mathematics Intermediate Technical

Examination time: Two hours and fifteen minutes (includes reading time)

Scan and upload time: Twenty minutes (at the end of the examination)

[Total time on examination countdown clock: Two hours and thirty-five minutes]

Total marks: 100

INSTRUCTIONS TO THE CANDIDATE

Failure to adhere to any of these instructions could incur examination sanctions including disqualification from the examination.

- 1. The question paper is only available on the ASSA Exam Platform as a PDF download and may not be printed.*
- 2. Your computer must be placed, and camera angled, so that your head and shoulders as well as your writing area on your desk is visible to the invigilator. Readjust your camera if you bump or move your computer by accident.*
- 3. You are required to write your answers on a clean A4 examination pad. Write only on one side of the paper and number your pages.*
- 4. Ensure that you have your exam permit handy. It reflects your candidate number to input **as part of the two hours 15 minutes examination**. Write your **full Candidate Number**, which starts with “A211-exa-2025-...” at the top of each page during the examination time only. **Do not use your name and/or member number anywhere on your answer script.***
- 5. The cell phone to be used to scan your final answer script must be switched **OFF** during the two hours and 15 minutes examination time. Place the cell phone out of reach.*
- 6. You are strongly encouraged to use the first 15 minutes as reading time only, however, you may start answering the paper whenever you are ready.*
- 7. Attempt all questions, beginning your answer to each question on a new page.*

8. *Write in black or dark blue pen.*
9. *Show calculations where appropriate. You may use blank paper to carry out rough work calculations. This rough work paper must not be scanned as part of your answer script. You may use a calculator from the ASSA-approved list only.*
10. *You may not access any file from your computer, use any other computer software (e.g., Email or Excel), or open any other browser, collaboration tools or AI tools (e.g. Copilot, ChatGPT etc.) during the examination. Nor may you use Grammarly, Grammarly Premium or similar add-ins. Microsoft Word spellcheck is allowed.*
11. *You are not permitted to bring any other material into the examination (e.g. a Formulae and Tables book). Any such information/material that may be required will be provided to you in the examination.*
12. *Mark allocations are shown in brackets.*
13. *Assume that months are all equal length, unless otherwise stated.*
14. *At the end of the two hours and 15 minutes examination time, you must stop writing and start scanning and uploading your script. **You may NOT continue to write during the 20 minutes of scanning and upload time.***
15. *Scan ALL your answer pages to PDF so that your candidate number is clear at the top of each page.*
16. *Save your PDF scanned file using your **full candidate number** as file name. **Do not name your answer script with your name or member number.***
17. *Transfer your scanned script file to your computer and upload it into the ASSA Exam Platform.*
18. *Click on the **Upload Answers** link below the examination paper link. Ensure you click on **Finish Attempt** below the upload box and again on **Finish All and Submit**, **before** the 20-minute scanning and upload time is up. (After submission the number of files successfully submitted will be reflected.)*

Note: The Actuarial Society of South Africa will not be held responsible for any late submissions or loss of data where candidates have not followed instructions as set out above. The onus remains on you to read and implement all examination instructions.

END OF INSTRUCTIONS

QUESTION 1

The effective half-yearly interest rate is 2.55%.

Calculate, using the above interest rate and showing all workings

- a. The annual force of interest.
- b. The annual nominal interest rate compounded every five years.
- c. $\frac{d^{(6)}}{6}$.
- d. $(I\ddot{a})_{\overline{10}|}$.

Express your answers for (a), (b) and (c) as percentages rounded to the nearest four decimals.

[8]

[Total 8]

QUESTION 2

You are given the following information:

- The current price of a four-year zero-coupon bond is R79 per R100 nominal.
- The current price of a three-year coupon-bearing bond is R92 per R100 nominal. The bond pays annual coupons at a rate of 3% per annum in arrear and is redeemable at par.
- $f_{2,2} = 0.05$
- $y_3 = 0.06$

- i. Calculate y_1, y_2 and y_4 .

[5]

- ii. Calculate the one-year effective forward rates: $f_{0,1}, f_{1,1}, f_{2,1}$ and $f_{3,1}$.

[5]

[Total 10]

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QUESTION 3

The force of interest, $\delta(t)$, is given by the formula $\delta(t) = 0.02$ at any time t ($t > 0$), where t is measured in years.

An amount of R1 is invested at time t , earning interest according to the above formula.

- i. State an expression (in terms of $\delta(t)$) for the present value of the payment at time zero, then simplify the expression as far as possible.

[1]

An annuity is payable continuously between time 0 and time t , where the rate of payment per unit time is constant and equal to R1.

- ii. State an expression (in terms of $\delta(t)$) for the present value of the annuity at time zero, then simplify the expression as far as possible.

[3]

A t -year annuity is payable continuously and increasing continuously. The rate of payment is s at time s , where $0 \leq s \leq t$.

- iii. Write down a formula for the present value of this annuity, at time zero, as a function of
- a level annuity payable continuously and
 - other terms.

[1]

A ten-year annuity is payable continuously. The annual rate of payment starts at R100 per annum at time zero and decreases continuously to R90 per annum at time ten.

- iv. Calculate the present value of this annuity at time zero.

[5]

[Total 10]

QUESTION 4

A firm has a portfolio of liabilities that requires it to pay R204,020 in exactly seven years' time and R980,402 in exactly eleven years' time.

The firm intends to immunise the liability portfolio against small changes in the interest rate by holding a portfolio of zero-coupon bonds. The firm purchases a five-year zero-coupon bond with a nominal value X and a 15-year zero-coupon bond with a nominal value Y .

The interest rate applicable to all assets and liabilities is currently 12.5% per annum effective.

- i. Calculate the discounted mean term of the portfolio of liabilities.

[3]

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- ii. Calculate the values of X and Y to the nearest Rand. [8]
 - iii. Describe how you would determine whether immunisation has been achieved. Define any notation used. [3]
- [Total 14]

QUESTION 5

Consider a mortgage loan of size L that is repayable annually in arrear over a period of n years. The instalment in the first year is P , in the second year it is $2P$, in the third year it is $3P$ and so on until the end of the loan contract. The annual effective interest rate is denoted by i and is positive.

- i. Derive an expression for the flat rate of interest on the loan in terms of L, P and n .
Hint: $\sum_{k=1}^n k = \frac{n(n+1)}{2}$ [3]
- ii. Derive an expression for the capital portion of the fifth instalment on the loan, using the prospective method. The expression should be in terms of P, i and annuity functions. [4]

Directly after the fourth instalment, the borrower decides to change their repayment schedule for the remainder of the term to level annual instalments of size X . The loan term remains unchanged.

- iii. Derive an expression for the new annual payment X in terms of P and annuity functions. [3]
- iv. Derive an expression for the capital portion of the fifth instalment on the loan, i.e. the first instalment after the loan was restructured, in terms of P and annuity functions. [2]
- v. Comment on the relative size of the capital portion, of the instalments, under the new arrangement by comparing your answers from parts ii. and iv. [2]
- vi. Explain, using your answers to parts i. and v., the effect of the new repayment schedule on the flat rate of interest of the loan. [3]

[Total 17]

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QUESTION 6

A fixed-interest bond with coupons of 10% per annum payable annually in arrear can be redeemed at par on any coupon-payment date n years after the date of issue. The value of n is to be selected by the borrower, subject to the restriction that $4 \leq n \leq 6$.

Investor A buys the fixed-interest bond at issue and is subject to 30% tax on income and 38% tax on capital gains. Investor A wants to earn a net effective yield of at least 7.5% per annum.

- i. Determine the value of n that the investor needs to assume when calculating the price for the bond to guarantee the net effective yield.

[3]

Investor B is subject to 25% income tax and 38% capital gains tax. Investor B wants to earn a net effective yield of at least 7.5% per annum.

- ii. Determine the value of n that the investor needs to assume when calculating the price for the bond to guarantee the net effective yield.

[3]

Investor A buys a nominal amount of R10,000 of the fixed-interest bond at issue.

The rate of inflation is 2% throughout the lifetime of the investment and Investor A wants to earn a real yield of at least 7.5% per annum.

- iii. Calculate the price Investor A should pay for this investment.

[10]

[Total 16]

QUESTION 7

A local municipality is planning to build a new suburb at a location close to a highway and major shopping centre. The aim is to attract new residents to the area, boost the local economy and build up municipal funds through higher taxes as a result.

The expected costs associated with the project are as follows:

- The initial cost of R100m is for purchasing plots in the area designated for development.
- At the start of each of the first three years: R75m, R50m, and R25m, are the respective costs for construction.
- Continuous outflow of maintenance and service fees are expected from the start of the fifth year of the project and payable in perpetuity. The initial rate of maintenance cost is estimated at R3m per annum. This will increase continuously at the rate of wage inflation, which is estimated at 4.5% per annum.

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The estimated income from the project is as follows:

- Income of R50m per annum in arrear during the first four years of the project results from the sale of empty plots.
- Continuous inflow of municipal rates and taxes from the start of the fifth year of the project and receivable in perpetuity. The initial rate of income is expected to be R15m per annum, increasing by R6m per annum, at the start of each of the following five years.

Thereafter, rates and taxes will be increased annually in arrear at the rate of the consumer price inflation, which is estimated to be 3.5% per annum. The first such increase will occur at the start of the 11th year of the project.

- Calculate the net present value of the project at 5.5% per annum effective. [19]
- Show that the discounted payback period of the project is before the end of the tenth year of the project. [4]

Ten years after the start of the project, the estimate of future consumer price inflation is updated from 3.5% to 2.5% per annum.

- Explain, with reason(s), how this would impact the calculated discounted payback period. [2]
- [Total 25]

[GRAND TOTAL 100]

END OF EXAMINATION