



Decoding CQS – An Ode to Fermat's Last Theorem

Feedback from SAM Phase II Sub-committee
presented by Edmond Vigoureux



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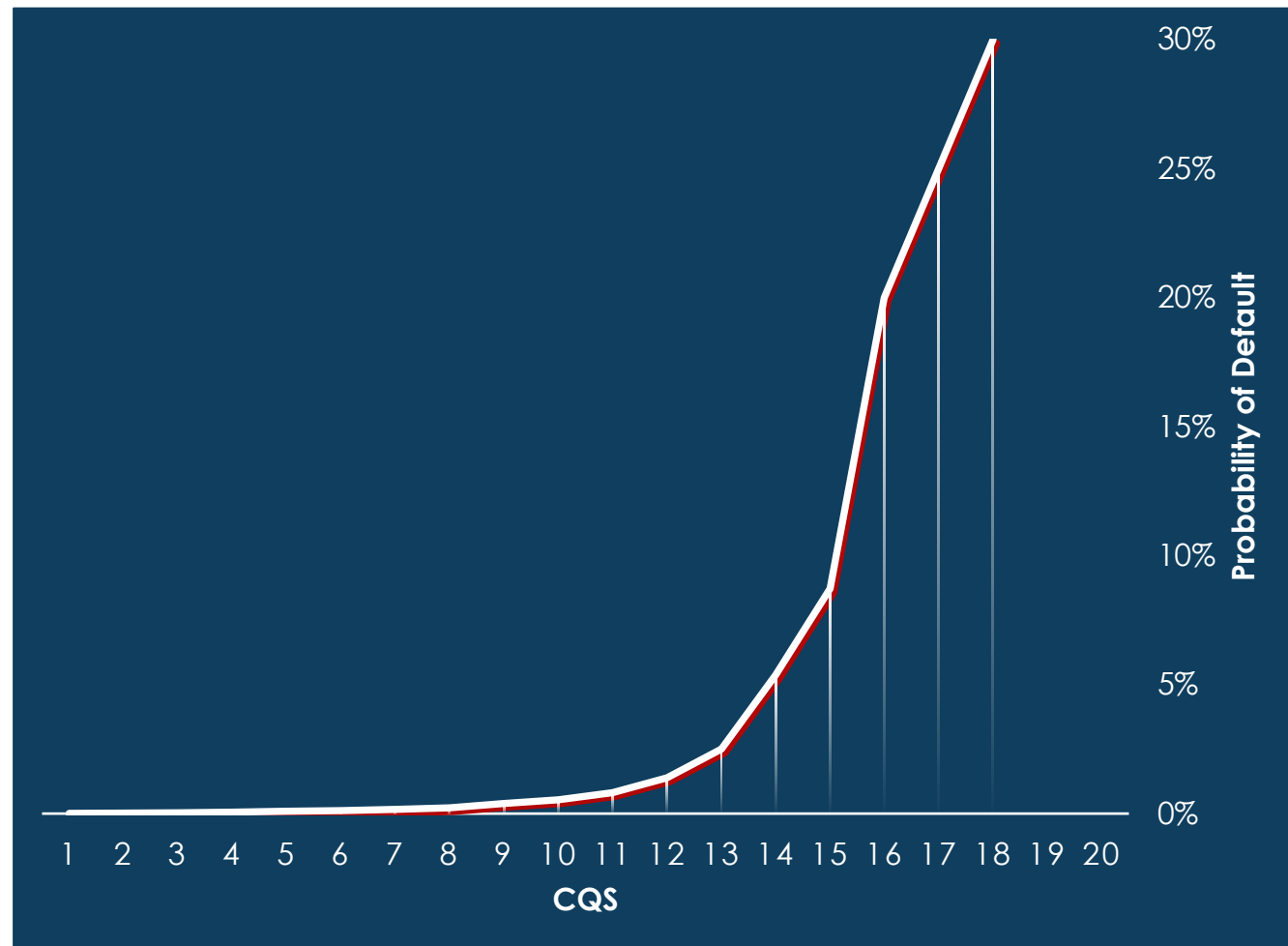
Advancing Actuarial Excellence

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CQS	PD*
1	0.01%
2	0.02%
3	0.03%
4	0.06%
5	0.09%
6	0.11%
7	0.16%
8	0.22%
9	0.39%
10	0.54%
11	0.81%
12	1.39%
13	2.50%
14	5.37%
15	8.72%
16	20%
17	25%
18	30%

*Probability of Default (Best Estimate)
as per FSI 4.1



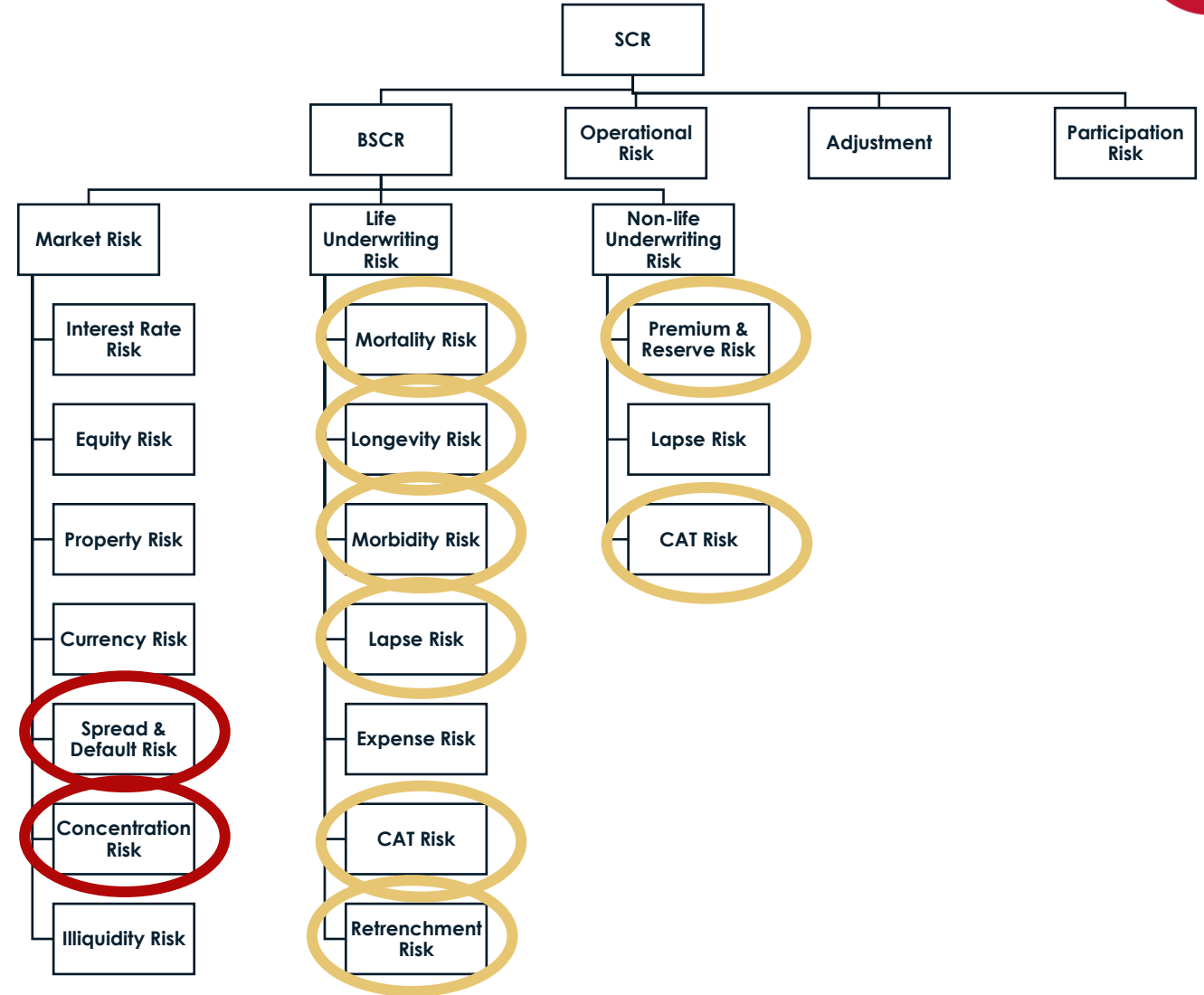


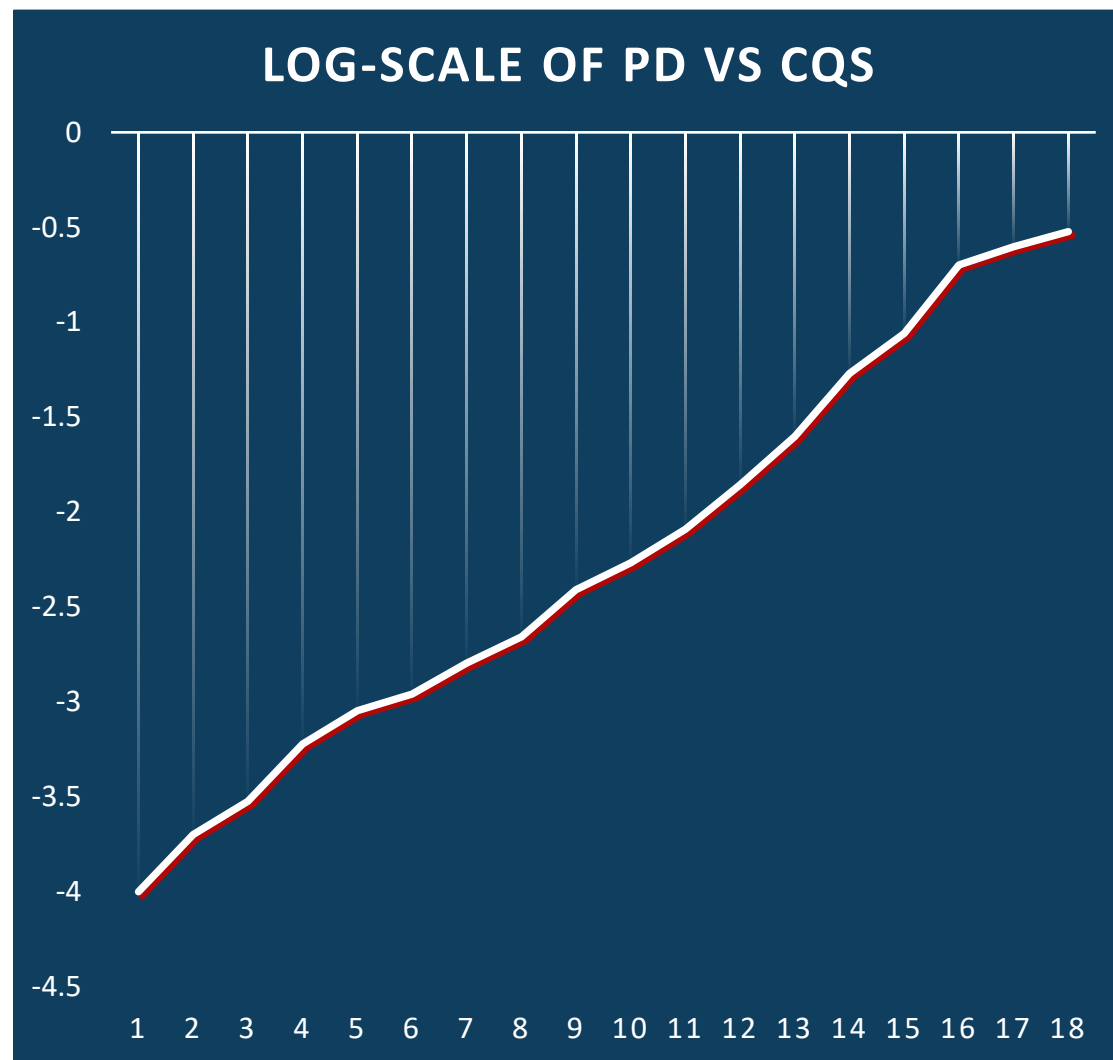
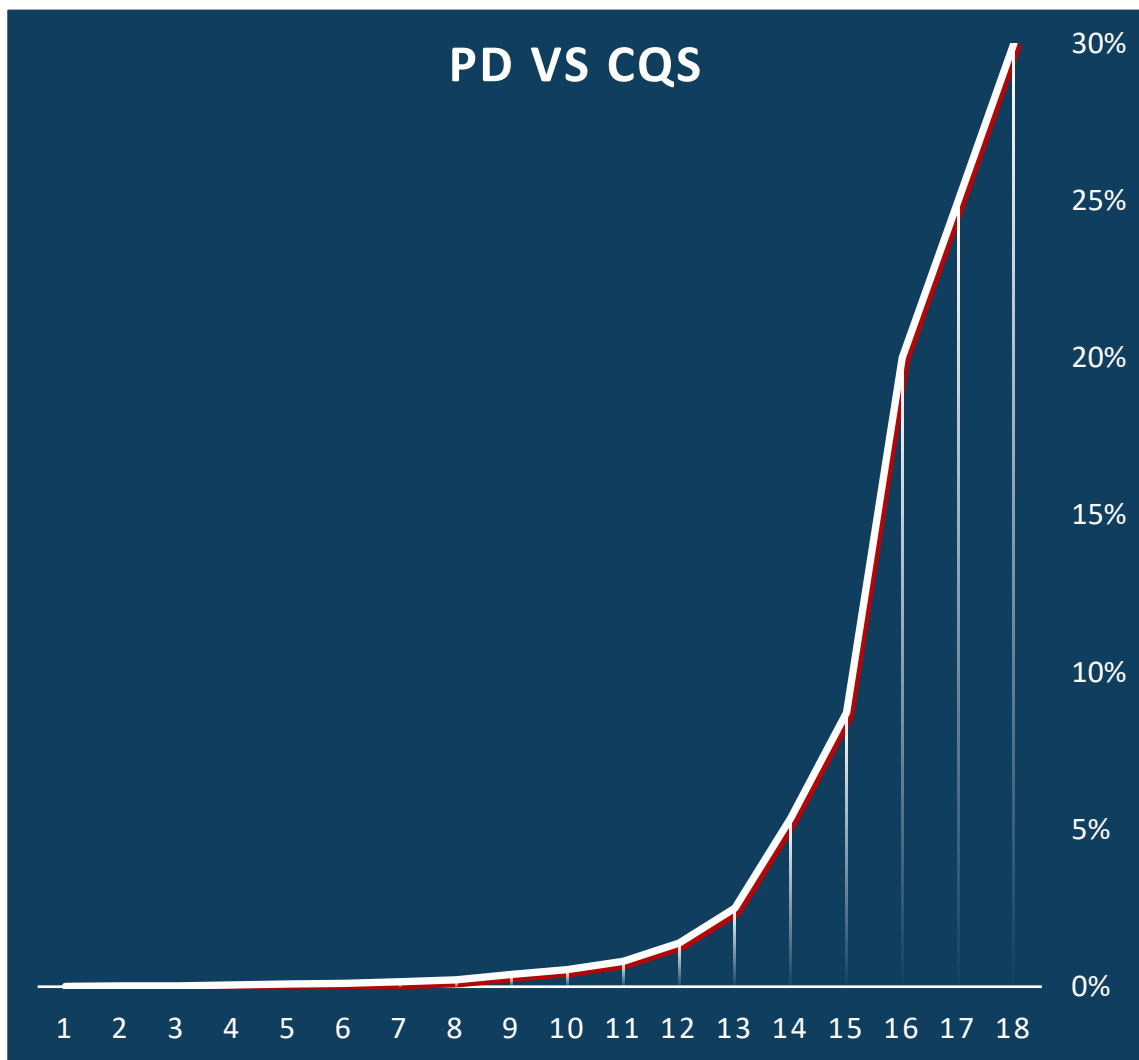
SAM Balance Sheet / BOF

RI Recoverables

Negative BEL for RI

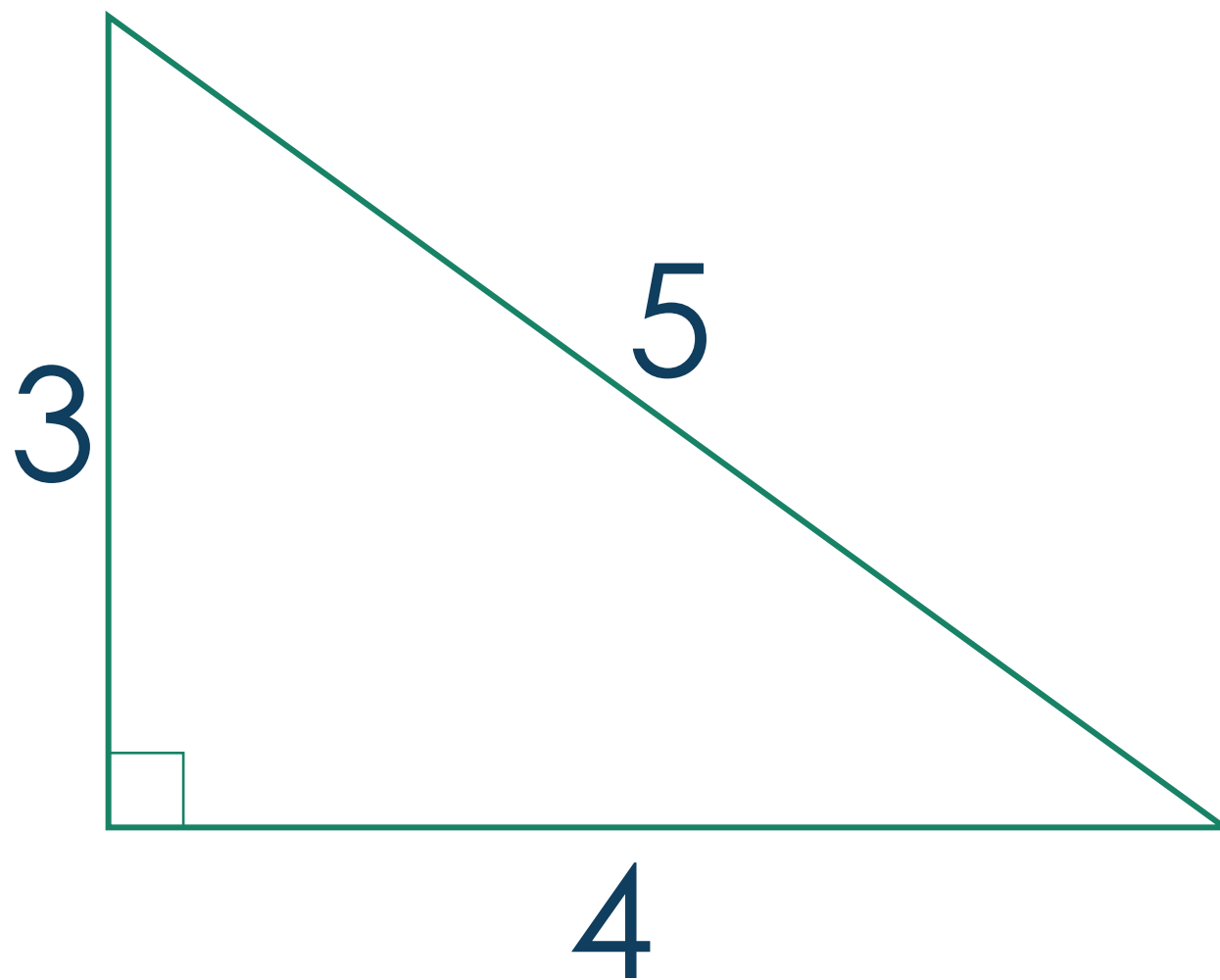
-  Default adjustment to BE
-  Applied directly through capital charge
-  Impairment of risk mitigating instruments

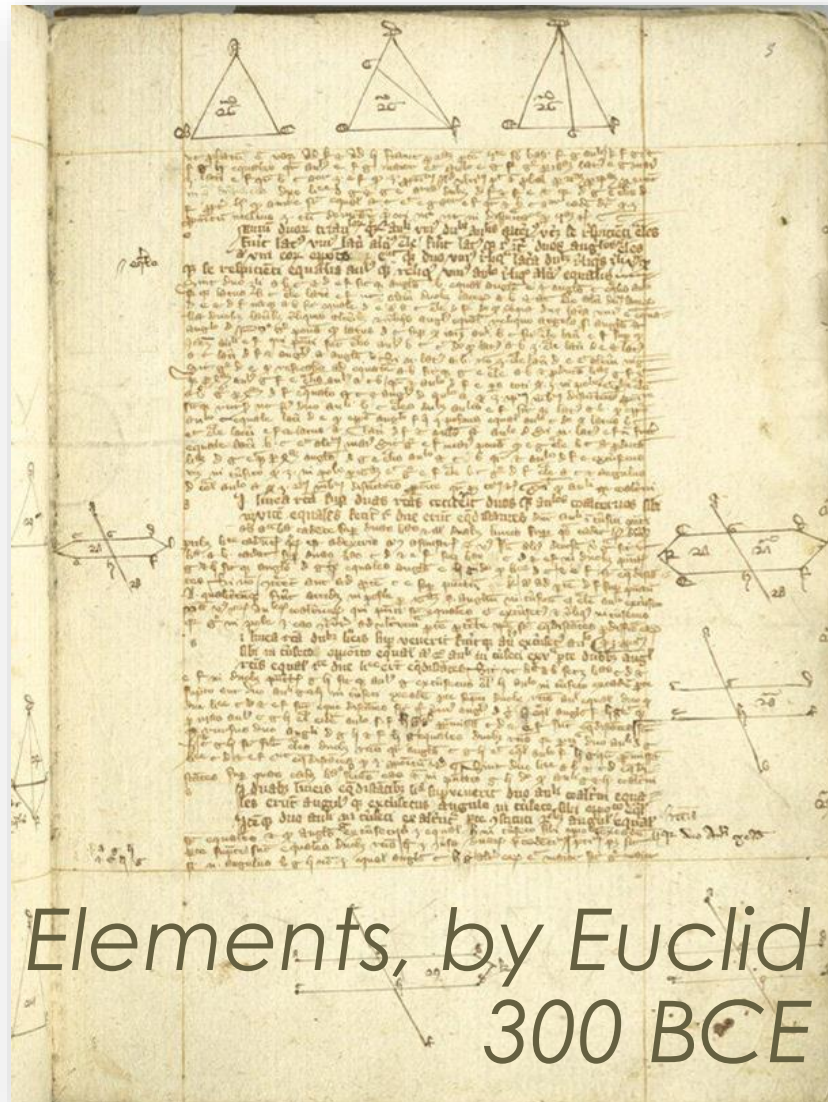






$$a^2 + b^2 = c^2$$





Elements, by Euclid
300 BCE

$$3^2 + 4^2 = 5^2$$

$$5^2 + 12^2 = 13^2$$

$$7^2 + 24^2 = 25^2$$

...

∞

QED

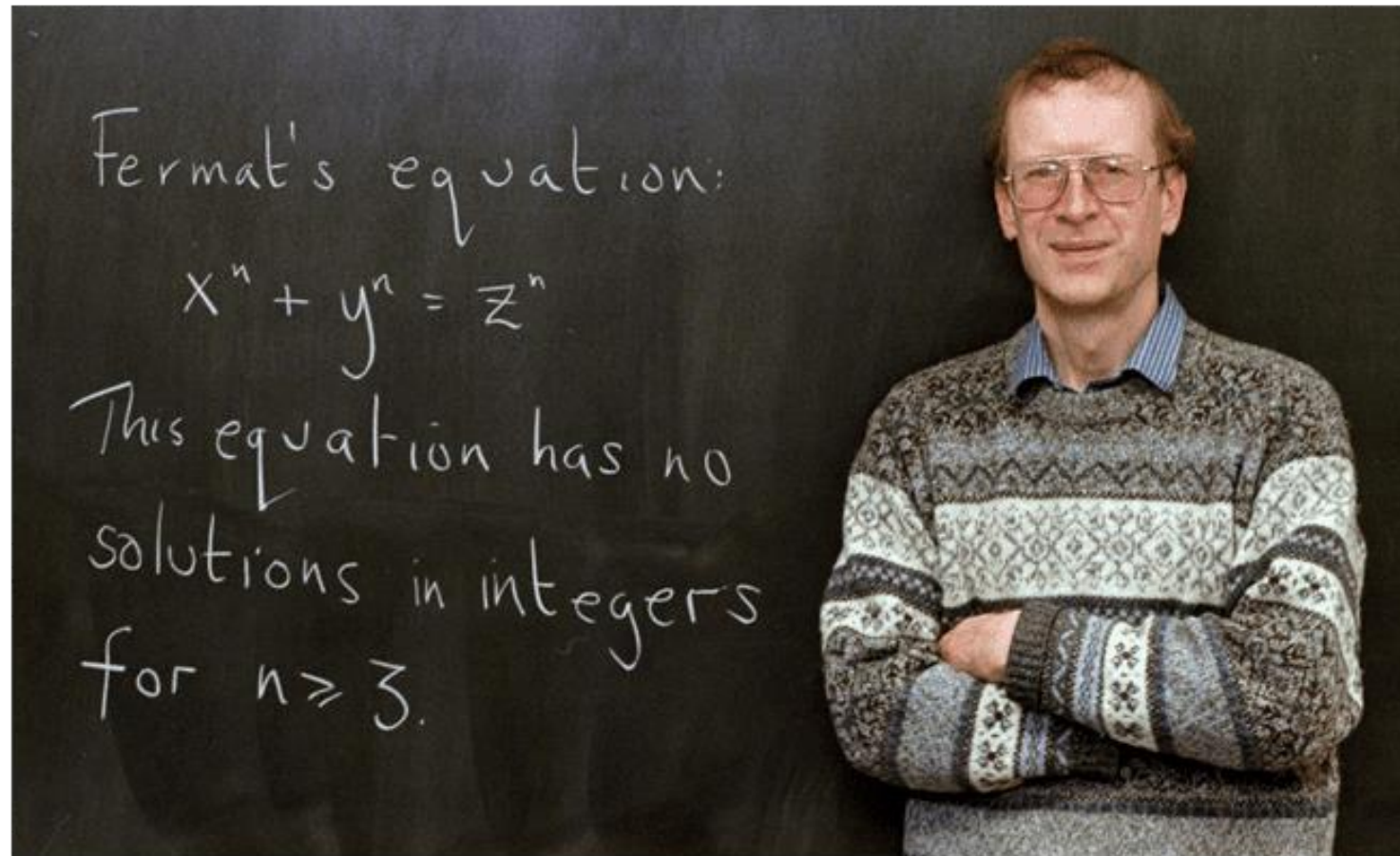


$$a^3 + b^3 = c^3$$

$$a^4 + b^4 = c^4$$

...

$$a^n + b^n = c^n, \forall n > 2$$



Andrew Wiles, proved in 1994



Arithmeticon Liber II.

61

interuallum numerorum 2. minor autem
1 N. atque ideo maior 1 N. + 2. Oportet
itaque 4 N. + 4. triplos esse ad 2. & ad-
huc superaddere 10. Ter igitur 2. adsci-
tis vnitatibus 10. æquatur 4 N. + 4. &
fit 1 N. 3. Erit ergo minor 3. maior 5. &
satisfaciunt quæstioni.

εἰ ἐνός. ὁ ἀρα μείζων ἔσται εἰ ἐνός μὲν β. δει-
σει ἀρα ἀριθμὸς δ' μονάδας δ' τριπλασίονας
ἔσθ' μὲν β. & ἐτι ὑπερέχειν μὲν γ. τρις ἀρα
μονάδας β' μὲν γ. ἴσται εἰσὶν εἰς δ' μονάσει
δ. καὶ γίνεται ὁ ἀριθμὸς μὲν γ. ἔσται ὁ μὲν ἐλάσ-
σαν μὲν γ. ὁ δὲ μείζων μὲν ε. καὶ ποιῶσι τὸ
πρόβλημα.

IN QUÆSTIONEM VII.

CONDITIONIS appositæ eadem ratio est quæ & appositæ præcedenti quæstioni, nil enim
aliud requirit quàm vt quadratus interualli numerorum sit minor interuallo quadratorum, &
Canones iidem hic etiam locum habebunt, vt manifestum est.

QUÆSTIO VIII.

PROPOSITUM quadratum diuidere
in duos quadratos. Imperatum sit vt
16. diuidatur in duos quadratos. Ponatur
primus 1 Q. Oportet igitur 16 — 1 Q. aqua-
les esse quadrato. Fingo quadratum a nu-
meris quotquot libuerit, cum defectu tot
vnitatum quod continet latus ipsius 16.
esto a 2 N. — 4. ipse igitur quadratus erit
4 Q. + 16. — 16 N. hæc æquabuntur vni-
tatibus 16 — 1 Q. Communis adiiciatur
vtrunque defectus, & a similibus auferan-
tur similia, fient 5 Q. æquales 16 N. & fit
1 N. 4. Erit igitur alter quadratorum 16.
alter verò 12. & vtriusque summa est 28. seu
16. & vterque quadratus est.

ΤΟΝ ὑπαρχόντα τετράγωνον διελὼν εἰς
δύο τετράγωνα. ἐπιτετέλεσθαι δὴ τὸ 16
διελὼν εἰς δύο τετράγωνα. καὶ τετέλεσθαι ὁ
πρότος δυνάμεις μίας. δέξει ἀρα μονά-
δας 16 λείψει δυνάμεις μίας ἴσται ἔσθ' τε-
τράγωνον. πλάσσω τὸ τετράγωνον ἀπὸ εἰς. ὅταν
δὴ ποτε λείψει ποσάτων μὲν ὅταν ἔσθ' ἢ τὸ 16
μὲν πλάσσω. ἔστω εἰς β' λείψει μὲν δ'. αὐτὸς
ἀρα ὁ πρῶτος δυνάμεις δ' μὲν 16
λείψει εἰς 16. ταῦτα ἴσα μόνον 16 λείψει
δυνάμεις μίας. κοινὴ προσκείμενα ἢ λείψει
καὶ ἀπὸ ὁμοίων ὁμοία. δυνάμεις ἀρα εἰ ἴσται
ἀριθμοὶ 16. καὶ γίνεται ὁ ἀριθμὸς 16. πέμπ-
των. ἔσται ὁ μὲν 28 εἰκοσπέμπτων. ὁ δὲ μὲν
εἰκοσπέμπτων. & οἱ δύο συντεθέντες ποιῶσι
τὸ εἰκοσπέμπτων, ἥτοι μονάδας 16. καὶ ἔσθ' ἐν ἑκάτερος τετράγωνον.

I have
discovere
d a truly
marvellou
s proof of
this, which
this
margin is
too
narrow to
contain.



Pierre de Fermat
French Mathematician
1601-1665



FSI 4.1

Assigning credit ratings for spread and default risk

- 9.9. The calculation of the capital requirements for spread risk and default risk relies on the assignment of credit ratings to instruments. The basis on which credit ratings are assigned in this Standard (and throughout the Financial Soundness Standards for Insurers) is through credit quality steps, which reflect long term historic probabilities of default, rather than external credit ratings. The use of credit quality steps allows for different external and internal ratings to be used in a consistent manner.
- 9.10. Where an insurer relies on external ratings to assign an instrument to the appropriate credit quality step and there is more than one external rating available, the historic default rates associated with each rating category must be assessed in order to assign the instrument to the most appropriate credit quality step. The credit quality step that is assigned should reflect the credit quality of the instrument and not the entity issuing the instrument.



S&P Ratings	CQS
AAA	?
AA+	?
AA	?
AA-	?
A+	?
A	?
A-	?
BBB+	?
BBB	?
BBB-	?
BB+	?
BB	?
BB-	?
B+	?
B	?
B-	?
CCC+	?
CCC	?
CCC-	?
CC	?
C	?

Moody's Ratings	CQS
Aaa	?
Aa1	?
Aa2	?
Aa3	?
A1	?
A2	?
A3	?
Baa1	?
Baa2	?
Baa3	?
Ba1	?
Ba2	?
Ba3	?
B1	?
B2	?
B3	?
Caa1	?
Caa2	?
Caa3	?
Ca	?
C	?

Fitch Ratings	CQS
AAA	?
AA+	?
AA	?
AA-	?
A+	?
A	?
A-	?
BBB+	?
BBB	?
BBB-	?
BB+	?
BB	?
BB-	?
B+	?
B	?
B-	?
CCC+	?
CCC	?
CCC-	?
CC	?
C	?



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Amended Mapping of Fitch Ratings' credit assessments under the Standardised Approach

Fitch Credit Rating	CQS
AAA	1
AA	1
A	2
BBB	3
BB	4
B	5
CCC	6
CC	6
C	6



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Amended Mapping of S&P Global Ratings' credit assessments under the Standardised Approach

S&P Credit Rating	CQS
AAA	1
AA	1
A	2
BBB	3
BB	4
B	5
CCC	6
CC	6
C	6



European Securities and
Markets Authority



EUROPEAN
BANKING
AUTHORITY



JOINT COMMITTEE OF THE EUROPEAN
SUPERVISORY AUTHORITIES

Amended Draft Mapping of Moody's Investors Service credit assessments under the Standardised Approach

Moody's Credit Rating	CQS
Aaa	1
Aa	1
A	2
Baa	3
Ba	4
B	5
Caa	6
Ca	6
C	6

Research Paper Contributors



Thank you -

Daniel Ponto

Jacques Bagraim

Kumaran Reddy

The CQS working group

Members of SAM Phase II Sub-Committee

Various colleagues in the industry

Proposed Methodology



By Credit Rating Agency and Credit Rating:

Step 1

Step 2

Step 3

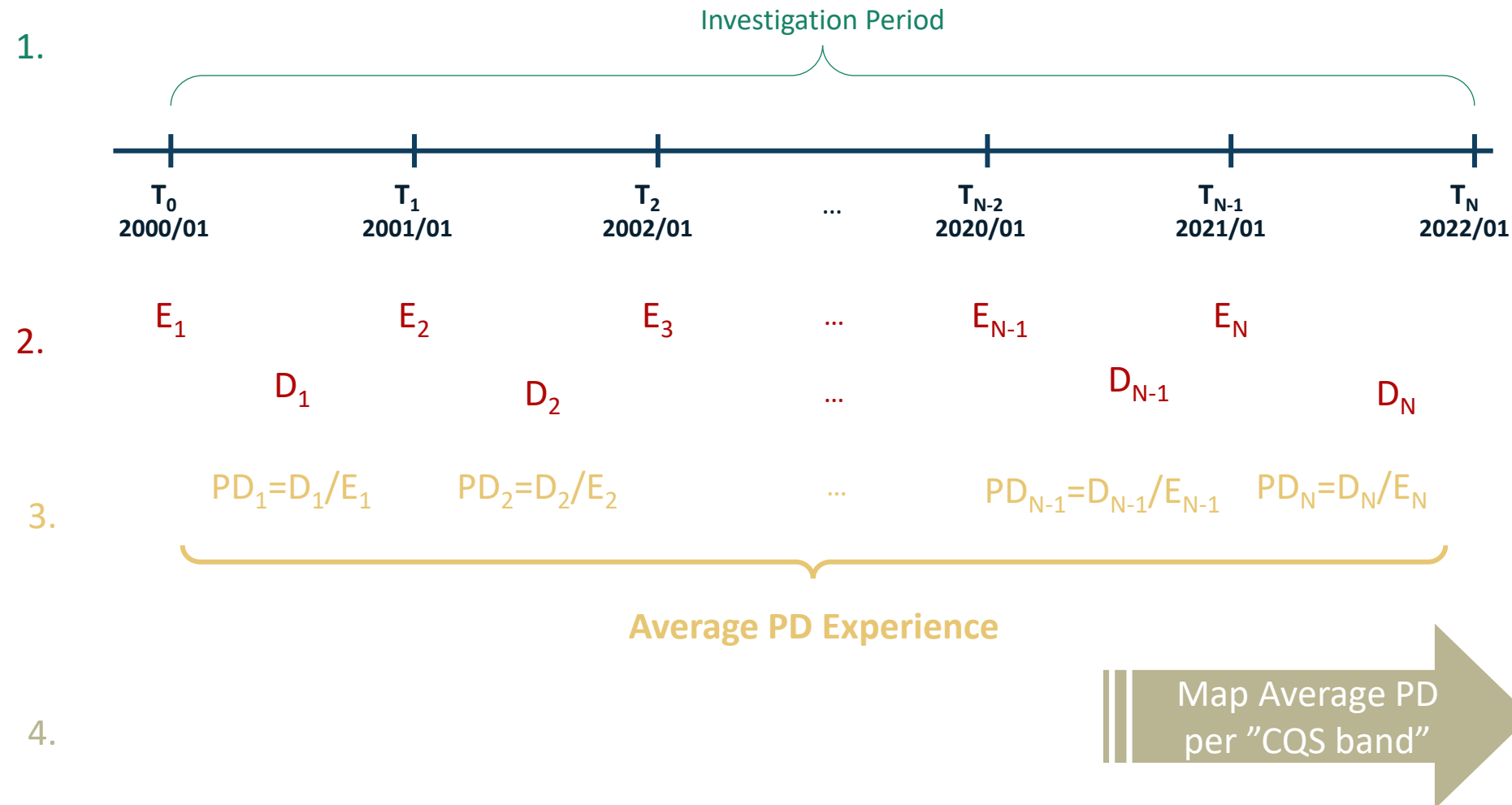
Initial Long Run Mapping:

The average historical experience over various investigation periods are considered, and an initial mapping is performed.

Proposed Methodology



Step 1 – Long-run Experience Mapping, for each credit rating:



CQS	PD	Band
1	0.01%	(0%; 0.014%)
2	0.02%	[0.014%; 0.024%)
3	0.03%	[0.024%; 0.042%)
4	0.06%	[0.042%; 0.073%)
5	0.09%	[0.073%; 0.099%)
6	0.11%	[0.099%; 0.133%)
7	0.16%	[0.133%; 0.188%)
8	0.22%	[0.188%; 0.293%)
9	0.39%	[0.293%; 0.459%)
10	0.54%	[0.459%; 0.661%)
11	0.81%	[0.661%; 1.061%)
12	1.39%	[1.061%; 1.864%)
13	2.50%	[1.864%; 3.664%)
14	5.37%	[3.664%; 6.843%)
15	8.72%	[6.843%; 13.206%)
16	20%	[13.206%; 22.361%)
17	25%	[22.361%; 27.386%)
18	30%	[27.386%; 100 %)

Proposed Methodology



By Credit Rating Agency and Credit Rating:

Step 1

Initial Long Run Mapping:

The average historical experience over various investigation periods are considered, and an initial mapping is performed.

Step 2

Short Run Tempering:

Trends and single-period experiences are considered against pre-agreed boundary / indicator levels. The mapping in the first step is then tempered or moderated accordingly.

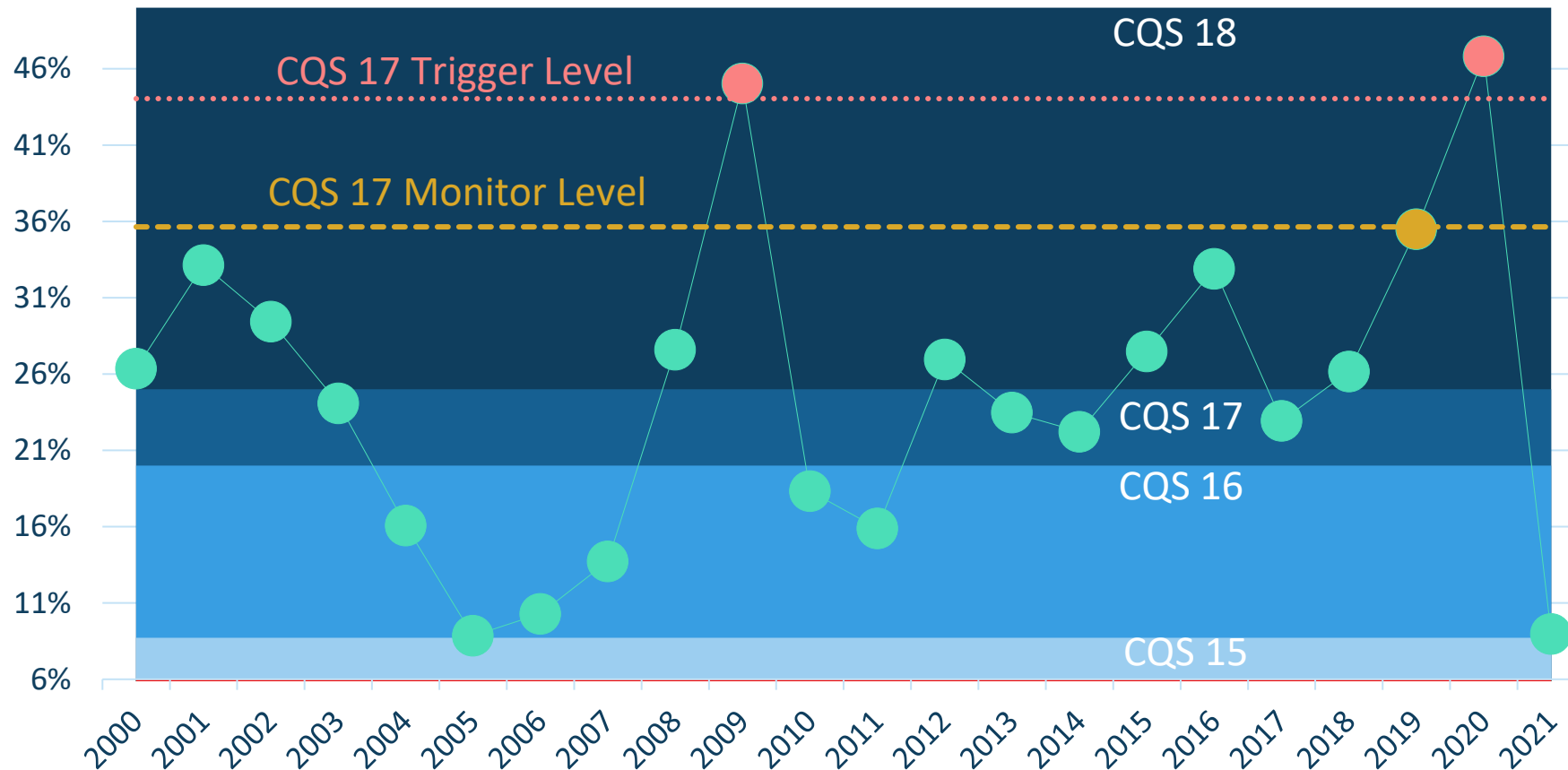
Step 3

Proposed Methodology



Step 2 – Short-run Experience Considerations

Historical Annual PD
for S&P CCC mapped to CQS 17 in Step 1



Proposed Methodology



By Credit Rating Agency and Credit Rating:

Step 1

Initial Long Run Mapping:

The average historical experience over various investigation periods are considered, and an initial mapping is performed.

Step 2

Short Run Tempering:

Trends and single-period experiences are considered against pre-agreed boundary levels. The mapping in the first step is then tempered or moderated accordingly.

Step 3

Final Considerations:

Do the mappings make sense when one considers it from higher perspective?

Proposed External Credit Rate Mapping



S&P Ratings	Proposed CQS Mapping
AAA	1
AA+	1
AA	2
AA-	2
A+	4
A	4
A-	4
BBB+	5
BBB	7
BBB-	8
BB+	8
BB	9
BB-	11
B+	13
B	14
B-	15
CCC+	17
CCC	18
CCC-	18
CC	18
C	18

Moody's Ratings	CQS Mapping
Aaa	1
Aa1	1
Aa2	1
Aa3	5
A1	5
A2	5
A3	5
Baa1	7
Baa2	8
Baa3	9
Ba1	10
Ba2	10
Ba3	12
B1	12
B2	14
B3	14
Caa1	15
Caa2	16
Caa3	16
Ca	18
C	18

Fitch Ratings	Proposed CQS Mapping
AAA	3
AA+	3
AA	3
AA-	4
A+	4
A	4
A-	4
BBB+	5
BBB	5
BBB-	8
BB+	8
BB	10
BB-	12
B+	13
B	13
B-	13
CCC+	17
CCC	18
CCC-	18
CC	18
C	18

Further Considerations



Further Considerations



Parental Guarantees and sub-exposures, e.g:



Further Considerations



Instrument vs Issuer ratings:

Reinsurance Default Adjustment

- Reinsurer rating used

Spread & Default Risk

- Both allowed
- LGD applied if the issuer rating is used

Concentration Risk

- Only issuer rating used

Further Considerations



Beware composite / derived ratings...

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Search Wikipedia

Search

Contents

hide

(Top)

Sovereign credit ratings

Short and long-term ratings

Corporate credit ratings

References

External links

Credit rating

41 languages

Article

Talk

Read

Edit

View history

Tools

From Wikipedia, the free encyclopedia

A **credit rating** is an evaluation of the **credit risk** of a prospective **debtor** (an individual, a **business**, **company** or a government). It is the practice of predicting or forecasting the ability of a supposed debtor to pay back the debt or **default**.^[1] The credit rating represents an evaluation from a **credit rating agency** of the qualitative and quantitative information for the prospective debtor, including information provided by the prospective debtor and other non-public information obtained by the credit rating agency's analysts.

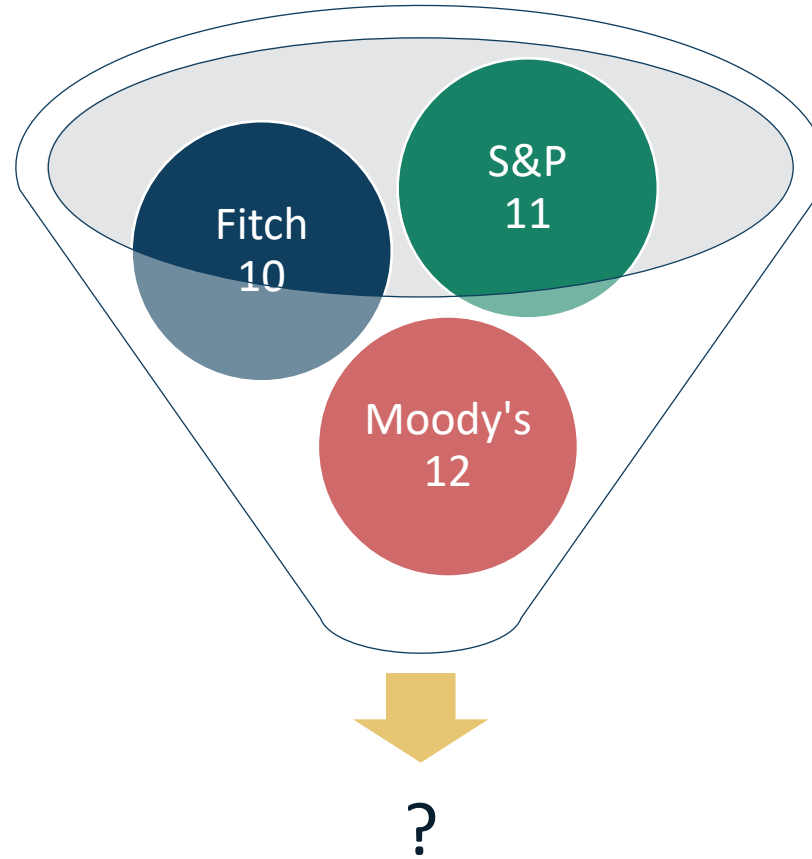
Credit reporting (or **credit score**) is a subset of credit rating. It is a numeric evaluation of an *individual's* credit worthiness, which is done by a **credit bureau** or **consumer credit reporting agency**.

Moody's		Standard & Poor's		Fitch Ratings		Rating description	EU Credit Quality Step ^{[18][19][20]}	Long run benchmark default rates (mid value) ^[21]	Short run benchmark default rates (trigger level) ^[21]
Long-term	Short-term	Long-term	Short-term	Long-term	Short-term				
Aaa	P-1	AAA	A-1+	AAA	F1+	Prime	1	0.1%	1.2%
Aa1		AA+		AA+					
Aa2		AA		AA					
Aa3		AA−		AA−					
A1	P-2	A+	A-1	A+	F1	Upper medium grade	2	0.25%	1.3%
A2		A		A					
A3		A−		A−					
Baa1	P-3	BBB+	A-2	BBB+	F2	Lower medium grade	3	1.0%	3.0%
Baa2		BBB		BBB					
Baa3		BBB−		BBB−					
Ba1	Not Prime	BB+	B	BB+	B	Non-investment grade speculative	4	7.5%	12.4%
Ba2		BB		BB					
Ba3		BB−		BB−					
B1		B+		B+					
B2	C	B	CCC+	B	C	Highly speculative	5	20%	35%
B3		B−		B−					
Caa1		CCC+		CCC+					
Caa2		CCC		CCC					
Caa3	CCC−	CCC−	Extremely speculative						
C	CC	CC							

Further Considerations



What happens if more than one credit rating agency rated an exposure?



Next Steps



- Where and how should this research be housed and allowed for in the industry?
 - Can more credit rating agencies be included in the study? (e.g. AM Best, GCR)
 - To cap or not to cap domestic / local exposures to the SA Sovereign CQS?
 - Are guidance needed on internal ratings?
 - Are more guidance needed in relation to CQS?
-



Thank you!

If you wish to get in touch with me:

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<end>

“Did you know?” & Next steps...



Fitch – the elephant in the room...

Prudential Communication 11 of 2022

Eligible institutions approved by the Prudential Authority (PA)

Objective of this Prudential Communication

This Prudential Communication serves to notify all banks, branches of foreign institutions, controlling companies and auditors of banks or controlling companies that the PA intends to derecognise Fitch Ratings (Fitch) as an eligible external credit assessment institution (ECAI).

1. Application

- 1.1 This Prudential Communication is applicable to all banks, branches of foreign institutions, and controlling companies, herein referenced to as banks.

2. Purpose

- 2.1 The purpose of this communication is to:

- 2.1.1 notify all banks that the PA intends to derecognise Fitch as an eligible ECAI. Therefore, ratings issued by Fitch may no longer be used in the calculation of regulatory capital requirements.

Current ECAI List:

- S&P
- Moody's
- GCR



FSI Technical Supervisory Observation, Topic 3:

This Topic discusses CQS and the expectations of the Prudential Authority from insurers in this regard. The technical derivation of the CQS is not in the scope of this Supervisory Observation.

A. Description

1. A CQS is similar, in nature, to ratings assigned by credit rating agencies and is derived in the context of the international scale local currency rating⁴ with 18 steps of descending credit quality.
2. The insurer's view of the historic long-term probability of default of a counterparty or asset should be reflected in the CQS, as described in paragraph 9.9⁵ of FSI 4.1 (Market Risk Capital Requirement).
3. A CQS also has an unrated category which should be used when an insurer has not yet assigned a CQS for the counterparty or asset. When an asset or counterparty is unrated, CQS 18 should be used when a particular Solvency Capital Requirement (SCR) factor table does not include the unrated CQS, e.g. calculating the variance of the loss distribution for type 1 exposures as set out in paragraph 9.18 of FSI 4.1. Some SCR modules might prescribe specific treatment of unrated assets or counterparties and insurers should adhere to those requirements.

B. How to assign a CQS

1. Technical derivation is not within the scope of this document, however, paragraph 9.10 of FSI 4.1 requires the insurer to assess the historic default rate and assign it to the appropriate CQS. This treatment and consideration by the insurer is in line with the principle that the board of directors must consider, and not automatically accept another entity's view of default probabilities.

Proposed Methodology



“The best estimate refers to the probability-weighted average of an insurer’s future cash-flows stemming from its insurance business, taking into account the time value of money and all possible scenarios of future potential outcomes”

Proposed Methodology



Step 1 – Long-run Experience Mapping

Per Credit Rating Agency and per Credit Rating - the average historical default experience is compared against CQS-PD table:

- Investigation period used – relevance vs sufficiency of data to allow a credible result

Aligned with SII as a benchmark

- S&P: Since 1 January 2000
- Moody's: Since 1 January 2000
- Fitch: Since 1 July 2021

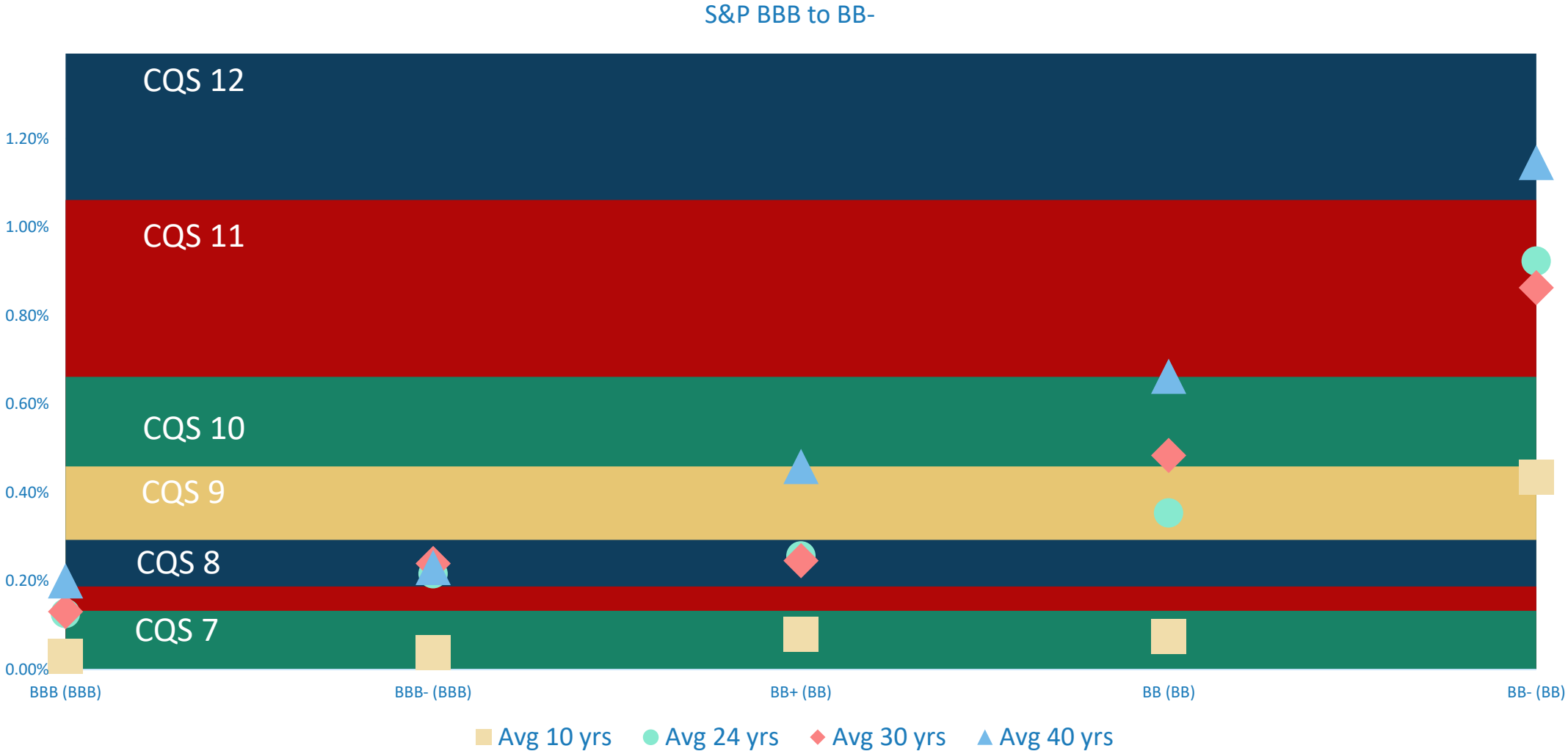
But do consider other periods

- Time-horizon: 1 year

- $$\text{Actual experience} = \frac{\# \text{ defaults that was rated at the start of the period}}{\# \text{ companies rated at the start of the period}}$$

CQS	BE PD (per FSI)	Band Used
1	0.01%	(0%; 0.014 %)
2	0.02%	[0.014%; 0.024%)
3	0.03%	[0.024%; 0.042%)
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17	25.00%	[22.361%; 27.386%)
18	30.00%	[27.386%; 100 %)

Proposed Methodology



Proposed Methodology



Step 2 – Short-run Experience Considerations

The mapping in step 1 is then tempered/moderated in this step by considering the short-run experience:

- The annual actual experience is considered over the investigation period
- Consideration is then given to whether it breaches two indicator levels

CQS	Monitor Level	Trigger Level
1	0.68%	1.14%
2	0.69%	1.15%
3	0.71%	1.15%
4	0.75%	1.17%
5	0.79%	1.19%
6	0.81%	1.21%
7	0.88%	1.24%
8	0.96%	1.28%
9	1.26%	1.62%
10	1.54%	1.96%
11	2.05%	2.57%
12	2.92%	3.56%
13	4.38%	5.17%
14	8.18%	9.32%
15	12.72%	14.61%
16	28.60%	35.00%
17	35.64%	44.04%
18	n/a	n/a